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ANALYTICAL MODELING FOR NON-LINEAR VIBRATION ANALYSIS OF FUNCTIONALLY GRADED PLATE SUBMERGED IN FLUID

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ABSTRACT

The present work proposes an analytical investigation of dynamic characteristics for non-linear vibration of functionally graded plates submerged in fluidic medium. The governing equation of the plate vibrating in fluid is analytically derived based on Kirchhoff's plate theory and the potential flow theory. The influence of fluidic medium is incorporated in governing equation in form fluids forces associated with inertial effects of its surrounding fluids. The velocity potential function and Bernoulli's equation are used to describe the fluid forces acting on plate surface. Both partially and totally submerged plate configurations are considered. The significance of using Berger formulation for in-plane forces is such that it introduces cubical nonlinearity to the system. The application of Galerkin's method reformulates the derived governing equation into well known Duffing equation. An approximate solution for nonlinear governing equation of coupled fluid-plate system is obtained by using a perturbation technique. For assessment of the present results, they are compared with the experimental and numerical results of isotropic submerged plate which shows good agreements. New results for fundamental frequencies as affected by level of submergence, fluid density and immersed depth of plate are presented for two different boundary conditions. The effect of above parameters on peak amplitude and frequency response curves is also established in the present study.

KEYWORDS: Non-linear Vibration, Classical Plate Theory, Fluid-plate Interaction, Virtual Added Mass.

In recent years the dynamic behavior of plate structures under fluidic medium has received considerable attention due to its wide applications in ship building, nuclear, ocean and naval engineering. It is well known that the vibrations of submerged structures are different than those in vacuum. Therefore, the study of dynamic characteristics of plate structures coupled with fluid is important for its safety and designing purpose. The vibration characteristics of submerged isotropic plate is rigorously treated and well studied in literature. The analytical approach for the vibration problem of coupled fluid-plate system was first initiated by Lamb, 2016. He calculated the change in natural frequency of a thin clamped circular plate in contact with water based on Rayleigh's method, and then Powell and Roberts, 1922 experimentally verified the theoretical results of Lamb, 2016. The dry and wet dynamic characteristics of cantilever plates partially or totally immerged in water are studied by Fu and Price, 1987. Kwak and Kim, 1991 determined the added virtual mass incremental (AVMI) factor which shows the increase in inertia due to presence of fluid. They studied the effect of fluidic medium on axisymmetric vibration of floating circular plate. Haddara and Cao, 1996 studied the dynamic behavior of rectangular plates vibrating under water. The effect of boundary conditions and depth of submergence has been investigated experimentally and analytically in their study. Soedel and Soedel, 1994 found the coupled equations of motion of plates carrying fluids. They developed a closed form solution for natural frequencies of fluid-plate coupled system. Kerboua et al., 2008 developed a mathematical model for free vibrating plate in contact with water using the combination of the finite element method and Sander's shell theory. Li et al., 2013 presented an analytical approach for the natural frequencies of a unidirectional vibrating steel strip partially submerged in fluid. Recently, Hosseini Hashemi et al., 2012 worked on free vibration analysis of horizontal rectangular plates partially and totally submerged in fluid. They developed a mathematical model for moderately thick rectangular plate based on the Mindlin plate theory for six different boundary conditions.

A few researchers worked on free vibration of thin functionally graded plates using classical plate theory. Natural frequencies of simply supported and clamped functionally graded plates were obtained by Abrate, 2008. Employing the von-Karman theory Woo et al., 2006 provided an analytical solution for the nonlinear vibration of FG square thin plates in vacuum. Similarly based on higher-order shear deformation theories and 3D methods the nonlinear free and forced vibration of functionally graded plates have also been investigated in vacuum by many researchers (Hosseini-Hashemi et. al., 2010, Talha and Singh, 2010 & Reddy and Cheng, 2003). To the best of author's knowledge, research studies on the dynamic behavior of submerged functionally graded plates have received very little attention. The present work addresses this by proposing an analytical approach for vibration of thin functionally

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graded plate coupled with fluid.

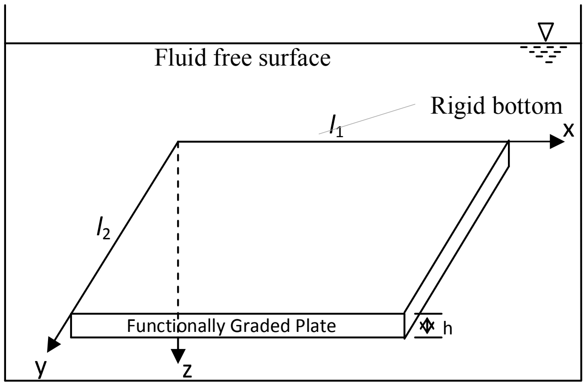

Figure 1: Plate Configuration

FUNCTIONALLY GRADED PLATE

A functionally graded (FG) plate under consideration is made up by mixing two distinct phases of metal and ceramic. It is assumed that material is graded only in the thickness direction. The top surface is made up of ceramic which is graded to metal by a power law distribution. The effective material properties of each layer for the FG plate can be estimated by Mori-Tanaka scheme and are represented as

$$E = \frac{9KG}{3K + G}$$

$$\frac{K - K_m}{K_c - K_m} = \frac{V_c}{1 + (1 - V_c)\frac{3(K_c - K_m)}{3K_m + 4G_m}}$$

$$\frac{G - G_m}{G_c - G_m} = \frac{V_c}{1 + (1 - V_c)\frac{(G_c - G_m)}{G_m + \frac{G_m(9K_m + 8G_m)}{6(K_m + 2G_m)}}}$$

$$\nu = \frac{3K - 2G}{2(3K + G)}$$

where K, G and E are the effective bulk modulus, shear modulus and Young's modulus respectively. K_m and G_m are the bulk modulus and shear modulus of the metal phase. K_c and G_c are the bulk modulus and shear modulus of the ceramic phase. The volume fractions of the metal and ceramic phases are related by $V_c + V_m = 1$. Where $V_c = (\frac{2z+h}{2h})^n$ and n is the gradient index also known as volume fraction exponent.

GOVERNING EQUATION OF PLATE COUPLED WITH FLUID

Based on Kirchhoff's thin plate theory, the classical form governing equation of an isotropic rectangular plate is rigorously treated in Ref. [Leissa, 1969]. Here using the similar approach a new governing equation is derived for a functionally graded plate considering the effect of surrounding fluid medium. To derive such a governing equation of coupled fluid-plate system as shown in Fig. 1, the following assumptions are considered in the modeling:

1. The plate is assumed as thin, perfectly elastic and homogenous made up of isotropic material and has a uniform thickness 'h' which is very small as compared to its other dimensions.
2. The normal stress σ_z acting in the transverse direction of plate is considered to be small and therefore, it is neglected from all stress-strain relationship while modeling.
3. Effects of shear deformation and rotary inertia are neglected.
4. The fluid flow is potential (i.e., homogeneous, incompressible, inviscid and its motion is irrotational).
5. Small amplitude of bending vibrations is considered (i.e., fluid motion is small).
6. Effect of boundary conditions on the plane wave number is neglected.

Based on above assumptions, the final version of the governing equation of functionally graded plate coupled with fluid can be expressed as

$$D_e\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) = -\rho_e h\frac{\partial^2 w}{\partial t^2} - \Delta P + N_x\frac{\partial^2 w}{\partial x^2} + N_y\frac{\partial^2 w}{\partial y^2} + P_z \quad (1)$$

Where, $D_e = \frac{E_e h^3}{12(1-\nu_e^2)}$ is the effective flexural rigidity of FG plate. E_e and ν_e are the effective modulus of elasticity and Poisson's ratio respectively. ρ_e is the effective density of the plate. ΔP is the fluid dynamic pressure difference on plate submerged partially or fully in fluid. w is the transverse deflection of the middle surface of plate. P_z is the transverse load per unit area, N_x, N_y are the in-plane or membrane forces per unit length in x and y directions whereas the in-plane shear force (N_{xy}) is neglected in the present modeling.

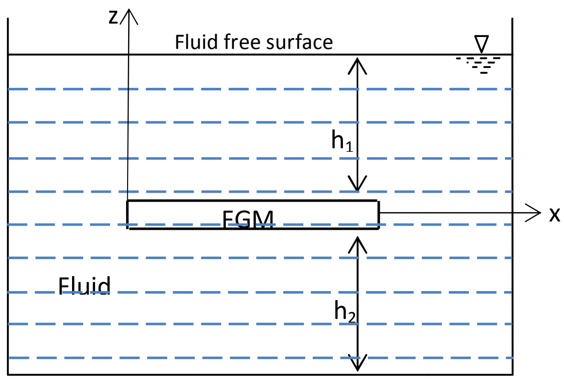

Figure 2: Horizontally submerged FG plate

The plate element for fluid dynamic pressure in case of horizontally submerged condition is shown in Fig. 2, Whereas Fig. 3 shows the plate in vertically immersed condition. In present study the fluid pressure acting upon the plate surface is expressed as a function acceleration which helps to form the required governing equation of a coupled fluid-plate system. The net fluid dynamic pressure difference (ΔP) for horizontally submerged plate can be stated as [Kerboua et. al., 2008]

$$\Delta P = P_{upper} - P_{lower} = -\frac{\rho_f}{\mu}\left[\frac{1+Ce^{2\mu h_1}}{1-Ce^{2\mu h_1}} - \frac{1+e^{-2\mu h_2}}{1-e^{-2\mu h_2}}\right]\frac{\partial^2 w}{\partial t^2} \tag{2}$$

Similarly, In case of a vertically immersed plate (as shown in Fig. 3.) the net fluid dynamic pressure difference can be expressed as [Li et. al., 2013]

$$\Delta P = P_{right} - P_{left} = -\frac{\rho_f}{\mu}\left[\frac{1+e^{2\mu d_1}}{1-e^{2\mu d_1}} - \frac{1+e^{-2\mu d_2}}{1-e^{-2\mu d_2}}\right]\frac{\partial^2 w}{\partial t^2} \tag{3}$$

Where, $c = \frac{g\mu-\omega^2}{g\mu+\omega^2}$ and ω is the natural frequency of plate in vacuum. ρ_f is the fluid density, μ is the plane wave number which represents the magnitude of wave motion and can be determined as $\mu = \pi\sqrt{\frac{1}{l_1^2}+\frac{1}{l_2^2}}$ for horizontally submerged plate and $\mu = \pi\sqrt{\frac{1}{l_1^2}+\frac{1}{Z_2^2}}$ for vertically immersed plate, Z_2 is the immersed depth of plate under fluid.

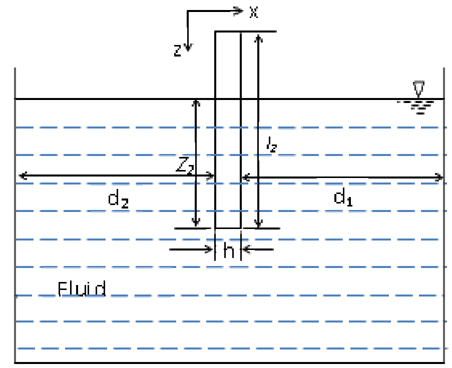

Figure 3: Vertically immersed FG plate

On substituting Eq. (2) and Eq.(3) into Eq.(1) we get a generalized equation of functionally graded plate coupled with fluid.

$$D_e\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) = -(\rho_e h + m_{add})\frac{\partial^2 w}{\partial t^2} + N_x\frac{\partial^2 w}{\partial x^2} + N_y\frac{\partial^2 w}{\partial y^2} + P_z \tag{4}$$

Where,

$m_{add} = -\frac{\rho_f}{\mu}\left[\frac{1+Ce^{2\mu h_1}}{1-Ce^{2\mu h_1}} - \frac{1+e^{-2\mu h_2}}{1-e^{-2\mu h_2}}\right]$ is the virtual added mass due to fluid for horizontally submerged plate and

$m_{add} = -\frac{\rho_f}{\mu}\left[\frac{1+e^{2\mu d_1}}{1-e^{2\mu d_1}} - \frac{1+e^{-2\mu d_2}}{1-e^{-2\mu d_2}}\right]$ is the virtual added mass due to fluid for vertically submerged plate.

SOLUTION OF GOVERNING EQUATION

The general solution for the transverse deflection of plate in terms of characteristic modal functions given by Galerkin's method can be stated as [Joshi et. al., 2015]

$$w(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{mn} X_m Y_n \psi_{mn}(t) \tag{5}$$

Where, X_m and Y_n are the characteristic or modal functions satisfying the boundary conditions of the plate in the fluidic medium, A_{mn} is an arbitrary amplitude and $\psi_{mn}(t)$ is time dependent modal term. Using the Berger's formulation the in-plane forces (N_x and N_y) are expressed in form of the middle surface strains and then expressing the middle surface strains in form of lateral deflection, the final form of in-plane forces can be written as

$$N_x = \frac{6D_e}{h^2 L_1 L_2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\int_0^{l_1}\int_0^{l_2}\left\{\begin{array}{l}\left(\frac{\partial X_m}{\partial x}\right)^2 {Y_n}^2 + \\ \nu_e\left(\frac{\partial Y_n}{\partial y}\right)^2 {X_m}^2\end{array}\right\} dx\, dy$$

$$A_{mn}{}^2\psi_{mn}(t)^2 \tag{6}$$

$$N_y = \frac{6D_e}{h^2L_1L_2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\int_0^{l_1}\int_0^{l_2}\left\{\begin{matrix}\left(\frac{\partial Y_n}{\partial y}\right)^2 X_m{}^2 + \\ \nu_e\left(\frac{\partial X_m}{\partial x}\right)^2 Y_n{}^2\end{matrix}\right\} dx\,dy \; A_{mn}{}^2\psi_{mn}(t)^2 \tag{7}$$

On substituting the Eq. (5), (6) and (7) into the Eq. (4), multiplying by X_m , Y_n on both side and then integrating over the whole plate area, one finds the governing equation of FG plate coupled with fluid as

$$\frac{(\rho_e h + m_{add})}{D_e}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}A_{mn}\int_0^{l_1}\int_0^{l_2}X_m{}^2Y_n{}^2\, dx\,dy\,\frac{\partial^2\psi_{mn}(t)}{\partial t^2}$$

$$+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}A_{mn}\,\psi_{mn}(t)\int_0^{l_1}\int_0^{l_2}\left\{\begin{matrix}(X_m{}^{iv}Y_n + 2\,X_m{}^{ii}Y_n{}^{ii} \\ +Y_n{}^{iv}X_m)\end{matrix}\right\} X_mY_n\,dx\,dy$$

$$-\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}A_{mn}{}^3\,\psi_{mn}(t)^3\int_0^{l_1}\int_0^{l_2}\left\{\begin{matrix}B_{1mn}X_m{}^{ii}X_m\,Y_n{}^2 + \\ B_{2mn}Y_n{}^{ii}Y_nX_m{}^2\end{matrix}\right\} dx\,dy = P_z \tag{8}$$

Where

$$B_{1mn} = \frac{6}{h^2L_1L_2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\int_0^{l_1}\int_0^{l_2}\left\{\left(\frac{\partial X_m}{\partial x}\right)^2Y_n{}^2 + \nu_e\left(\frac{\partial Y_n}{\partial y}\right)^2X_m{}^2\right\}dx\,dy$$

$$B_{2mn} = \frac{6}{h^2L_1L_2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\int_0^{l_1}\int_0^{l_2}\left\{\left(\frac{\partial Y_n}{\partial y}\right)^2X_m^2 + \nu_e\left(\frac{\partial X_m}{\partial x}\right)^2Y_n{}^2\right\}dx\,dy$$

The modal peak amplitude A_{mn} is normalized to unity and point load P_z is acting on the plate at position $(x_0\,, y_0)$. The Eq. (8) may be expressed as well known Duffing equation containing a cubical nonlinear term.

$$M_{mn}\frac{\partial^2\psi_{mn}(t)}{\partial t^2} + K_{mn}\,\psi_{mn}(t) + G_{mn}\,\psi_{mn}(t)^3 = P_{mn} \tag{9}$$

Where

$$M_{mn} = \frac{(\rho_e h + m_{add})}{D_e}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}A_{mn}\int_0^{l_1}\int_0^{l_2}X_m{}^2Y_n{}^2\,dx\,dy$$

$$K_{mn} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}A_{mn}\int_0^{l_1}\int_0^{l_2}\left\{\begin{matrix}(X_m{}^{iv}Y_n + 2\,X_m{}^{ii}Y_n{}^{ii} \\ +Y_n{}^{iv}X_m)\end{matrix}\right\} X_mY_n\,dx\,dy$$

$$G_{mn} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}A_{mn}{}^3\int_0^{l_1}\int_0^{l_2}\left\{\begin{matrix}-B_{1mn}X_m{}^{ii}X_m\,Y_n{}^2 \\ -B_{2mn}Y_n{}^{ii}Y_nX_m{}^2\end{matrix}\right\}dx\,dy$$

$$P_{mn} = \frac{P_0(t)}{D_e}Q_{mn}$$

Where, $Q_{mn} = X_m(x_0)Y_n(y_0)$, $X_m(x_0)$ and $Y_n(y_0)$ are given by the integration of delta function $\int_{-\infty}^{\infty}X_m(x)\,\delta(x-x_0)$ and $\int_{-\infty}^{\infty}Y_m(y)\,\delta(y-y_0)$ respectively.

On dividing M_{mn} on both sides of Eq. (9) and considering the system is under the influence of weak classical linear viscous damping μ_o then the nonlinear governing equation can be restated as

$$\frac{\partial^2\psi_{mn}(t)}{\partial t^2} + 2\mu_o\frac{\partial\psi_{mn}(t)}{\partial t} + \omega_{mn}^2\psi_{mn}(t) + \delta_{mn}\psi_{mn}(t)^3 = \frac{\lambda_{mn}P_0(t)}{D_e} \tag{10}$$

Where,

$$\omega_{mn}^2 = \frac{K_{mn}}{M_{mn}}$$

$$\delta_{mn} = \frac{G_{mn}}{M_{mn}}$$

$$\lambda_{mn} = \frac{Q_{mn}}{M_{mn}} = \frac{X_m(x_0)Y_n(y_0)}{M_{mn}}$$

ω_{mn} is the natural frequency of the plate vibrating in fluid and δ_{mn} is the cubic nonlinearity of plate.

FREQUENCY RESPONSE AND PEAK AMPLITUDE

The method of multiple scales is one of the most reliable perturbation techniques used for determining the frequency response equation from the Duffing equation and is well known presented in the book, 'Perturbation Theory and Methods' by Murdock. Authors in Ref. [Joshi et. al., 2015 & Murdock, 1999] have used this method to find the approximate analytical solution of Duffing equation for an isotropic rectangular plate. In present study, similar approach has been used to understand the nonlinear behavior of functionally graded plate under influence of fluid medium. The final expression for frequency response of the functionally graded plate coupled with fluid can be written as

$$e_{mn} = \frac{3\,\delta_{mn}J^2}{8\omega_{mn}} \pm \sqrt{\left(\frac{\lambda_{mn}^2}{4\,\omega_{mn}^2 {D_e}^2 J^2} P_0^2 - \mu_o^2\right)} \tag{11}$$

Where, e_{mn} is the detuning parameter which describes quantitatively the closeness of excitation frequency θ_{mn} with the fundamental frequency ω_{mn}, J is the amplitude of response and P_o is the amplitude of excitation force. The peak amplitude of response is found to be independent of cubic nonlinearity and is given by

$$J_p = \frac{\lambda_{mn}}{2\omega_{mn}D_e\mu_o} P_0 \tag{12}$$

RESULT AND DISCUSSION

This section presents the new results for first mode of natural frequency in terms of non dimensional frequency parameter of a functionally graded plate coupled with fluid medium. The results are presented for submerged plate for two configurations; (i) Horizontally submerged plate (ii) Vertically immersed plate. For validation of the proposed model a comparison study of non dimensional frequency parameters obtained by MPT [Hosseini-Hashemi et. al., 2012] and the present method (CPT) is conducted in Table 1 for intact isotropic plate submerged partially or totally in fluid with two different boundary conditions. The mechanical and geometrical properties of plate are taken as: Young's modulus E = 207 GPa, material density ρ = 7850 kg-m^{-3}, Poisson's ratio ν = 0.3, l_1 = 2 m, l_2 = 1 m and thickness h = 0.1 m. The dimensions of fluid tank are taken as 5 m x 5 m x 5 m. From Table 1 it is seen that, the present results are in agreement with the published one. The frequency parameters for different fluid levels $\left(\frac{h_1}{l_1}\right)$ obtained by present theory are slightly higher than the results of existing (MPT) theory it is because of ignoring the effects of shear deformation and rotary inertia.

Table 1: Comparison of non dimensional frequency parameter $\left(\omega_{mn} l_1^2 \sqrt{\rho h/D}\right)$ of intact plate submerged in water as a function of fluid level.

B.C.	In vacuum		In water			
			$\frac{h_1}{l_1} = 0$		$\frac{h_1}{l_1} = 0.1$	
	Present theory	MPT	Present theory	MPT	Present theory	MPT
S-F-S-F	9.86	9.45	7.33	6.71	6.88	6.33
S-S-S-S	49.34	48.30	42.27	41.42	39.22	38.46

Table 2: Comparison of frequency parameter of FG plate for the three boundary conditions (n = 1)

B.C.	Frequency parameter $\left(\omega_{mn} l_1^2/h\sqrt{\rho_c/E_c}\right)$			
	$(l_1/l_2) = 1$		$(l_1/l_2) = 2$	
	Present	Ref. [Joshi et. al., 2015]	present	Ref. [Joshi et. al., 2015]
S-S-S-S	3.461	3.460	8.653	8.651
C-C-S-S	4.972	4.970	12.961	12.955
C-C-F-F	0.899	0.897	2.585	2.577

The validation of present model for functionally graded plate in absence of fluid medium is carried out in Table 2. It shows the comparison of frequency parameters obtained using present model with the existing results. The properties of FG plate is taken from Ref. [Joshi et. al., 2015]. It is observed that the present results are in good agreement with published results.

The new results for frequency parameter of functionally graded plate coupled with fluid as affected by level of submergence, immersed depth of plate and gradient index are given in Table 3 and 4. The functionally graded material properties are taken from Refs. [Joshi et. al., 2015][Natarajan et. al., 2011] for silicon nitride and stainless steel (Si_3N_4/SUS304). The plate dimensions are l_1 = l_2 = 1 m and thickness h = 0.01 m. The fluid is taken as water with density 1000 kg-m^3 and damping factor 0.061. The dimensions of tank are assumed to be 5 m x 5 m x 5 m. Four boundary conditions are considered for the analysis. A point load of 10N, acting at point x_o = 0.375, y_o = 0.75 is considered for frequency response and peak amplitude. From Table 3 and 4, it is seen that regardless of boundary conditions, the frequency parameter decreases with increase in level of submergence (h_1/l_1) and immersed depth of plate (Z_2). This is because of the added virtual mass of plate due to surrounding fluid. It is also seen, for fixed level of submergence and immersed depth, as the gradient index (n) increases the frequency parameter decreases for both the boundary conditions.

Table 3: Non dimensional frequency parameter of functionally graded plate for various level of submergence and gradient index.

Frequency parameter $\left(\omega_{mn} l_1^2 / h\sqrt{\rho_c/E_c}\right)$					
B.C.	n	In vacuum	In water		
			$\frac{h_1}{l_1} = 0.1$	$\frac{h_1}{l_1} = 0.2$	$\frac{h_1}{l_1} = 0.3$
S-S-S-S	0	5.935	1.561	1.429	1.370
	1	3.46	1.303	1.201	1.154
	5	2.477	1.108	1.026	0.989
C-C-S-S	0	8.524	2.241	2.052	1.968
	1	4.970	1.871	1.724	1.657
	5	3.557	1.591	1.474	1.420

Table 4: Non dimensional frequency parameter of functionally graded plate as a function of immersed depth and gradient index.

Frequency parameter $\left(\omega_{mn} l_1^2 / h\sqrt{\rho_c/E_c}\right)$					
B.C.	n	In water			
		$Z_2 = 0.1$	$Z_2 = 0.2$	$Z_2 = 0.3$	$Z_2 = 0.4$
S-S-S-S	0	3.097	2.370	2.010	1.791
	1	2.332	1.885	1.636	1.477
	5	1.849	1.547	1.365	1.244
C-C-S-S	0	4.448	3.404	2.887	2.573
	1	3.349	2.707	2.350	1.121
	5	2.656	2.221	1.961	1.787

By employing the method of multiple scales, the peak amplitude (J_p) of the functionally graded plate vibrating under fluid is examined in present study. Table 5 shows the peak amplitude of the horizontally submerged plate for different level of submergence. It is observed from Table 5, the increase in level of submergence (h_1/l_1) decreases the peak amplitude of submerged plate. The reason being the resistance offered by the fluid medium to the vibratory motion of plate.

Table 5: Peak amplitude (mm) for functionally graded plate horizontally submerged in water (n =1).

Peak amplitude				
B.C.	In vacuum	In water		
		$\frac{h_1}{l_1} = 0.1$	$\frac{h_1}{l_1} = 0.2$	$\frac{h_1}{l_1} = 0.3$
S-S-S-S	9.688	3.647	3.361	3.231
C-C-S-S	13.850	5.214	4.804	4.619

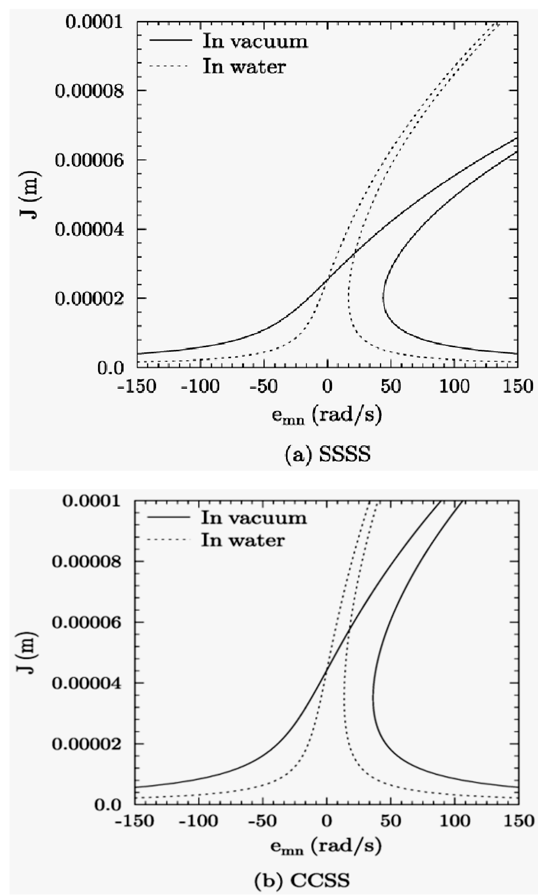

Figure 4: Non linear response curves of FG plate at different surrounding medium, $l_1 = l_2 = 1$, $h_1/l_1 = 0.1$.

In present study the geometrically nonlinear ($\delta_{mn} < 0$, soft spring and $\delta_{mn} > 0$, hard spring) response curves are plotted for given damping and excitation to study the phenomenon of bending hardening/softening of coupled fluid-plate system. The variations in response curve with different surrounding mediums (air and water) are given in Fig. 4. It is observed that the presence of fluid medium decreases the non-linearity of vibrating plate for both the boundary conditions. This phenomenon is called softening of hard spring i.e. δ_{mn} decreases due to effect of fluid medium but it remains positive. It is also seen that the non-linearity is more for SSSS submerged plate as compare to CCSS.

CONCLUSION

In this work, an attempt has been made to develop an analytical model for functionally graded plate coupled with fluid. The governing equation of coupled fluid-plate system is derived by combining classical thin plate theory and potential flow theory. The influence of the fluid medium is incorporated in governing equation in the form of surrounding fluid dynamic pressure. The velocity potential and Bernoulli's equation are employed to express the fluid dynamic pressure acting on plate element. The results obtained from present study shows the influence of boundary conditions, level of submergence, immerged depth and gradient index on vibration characteristics of partially and fully submerged FG plates. The frequency response curves with effect of cubic nonlinearity are presented using method of multiple scales. It is observed that the submergence decreases the non-linearity. Thus it can be concluded that the presence of surrounding fluid medium affects the vibration characteristics. The present model has obvious advantage of being ease of physical understanding, efficient computation time, and ease of parametric study. The presented approach can also be used for making analytical model of any curved structures subjected to random fluid dynamic pressures. For example turbine blades under influence of random fluid pressure induced by turbulent flow.

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