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STUDY OF A NONUNIFORMLY HEATING VISCOELASTIC FLUID IN TWO DIMENSIONS OVER A SLIPPING STRETCHING SHEET

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ABSTRACT

This article presents the results of our study into the flow of an incompressible viscoelastic fluid over a uniformly extending surface sheet under slip boundary conditions in the presence of porous media. Similarity transformation variables are used to convert the governing partial differential equations for the fluid flow to ordinary differential equations. Finally, the homotopy analysis method is used to semi-numerically solve the altered ordinary differential equations (HAM). As opposed to previous research, ours takes into consideration viscous dissipation, which is crucial in the case of optically transparent materials, and analyses the effect of the elastic parameter in detail. Through the use of graphs, we are able to examine the novel effects of parameters like the Eckert number, porous medium parameter, and velocity slip parameter, all of which have an impact on the flow and heat transfer. The convergence analysis of the suggested approach is discussed as well. For the purpose of verification, the current study is also compared to the previously published work, and a remarkable degree of agreement is discovered.

KEYWORDS: Uniformly, Eckert Number, Convergence

Because of its relevance to real-world problems like hot rolling, ber plating, and porous lubrication, research into fluid flow across a stretching surface has been particularly active. For the first time, Crane (2010) provided an analytical solution for the flow in a Newtonian boundary layer caused by a stretching surface. Newtonian fluid in laminar flow was investigated by Vleggar (2007) on a continuously accelerating stretched surface. Using a homogeneous heat flux, Dutta *et al.*, (2015) studied the motion of the temperature field caused by a stretching sheet. Many scholars (Pop and Na, 2008) (Ali, 2005) have studied a similar issue of Newtonian fluid flow because of the stretching surface.

The study of viscoelastic fluids across a stretching surface has many practical applications in engineering fluid mechanics. This includes the production of inks, paints, jet fuels, polymer extrusion, and the drawing of plastic fibre and wire. Since this class of fluid is finding more and more uses, many scientists have devoted their attention to it. For their research, Vajravelu and Rollins (2011) studied the influence of heat transport on a viscoelastic fluid across a stretching surface. The impact of MHD on the flow of a viscoelastic fluid caused by a stretching surface was studied by Andersson (1992). Suhas and Veena (2008) looked into the incompressible flow of viscoelastic fluid and heat transmission through a stretching sheet embedded in a porous media. Sanjaya and Khan (2016) talk about viscoelastic boundary layer fluid flow and heat transfer across an exponential stretching sheet. For his research, Nandeppanavar *et al.*, (2018) examined the flow and heat transfer properties of a viscoelastic fluid through an impermeable stretched sheet imbedded in a porous medium with viscous dissipation and heat transmission.

Velocity slip, a crucial issue in fluid mechanics, was not taken into account in the aforementioned research. Navier first mentioned it (2003). The incompressible flow near a solid surface under arbitrary boundary conditions was investigated by Thompson and Troian (2007). Investigated the slip effects and heat transfer analysis of a viscous fluid across an oscillating stretching surface. Studied MHD slip flow of a viscoelastic fluid across a stretched surface. Discussed the impact of slip and MHD on viscoelastic convection flow in a vertical channel. Examined slip flow in Newtonian and viscoelastic fluids. Slip conditions on the peristaltic flow of a Jeffrey fluid with a Newtonian fluid is examined. Analysed magnetohydrodynamic mixed convection viscoelastic slip flow through a porous material in a vertical porous channel with thermal radiation.

Heat transfer analysis is a vital tool in many areas of engineering, including the processing of food, the flow through filter media, and the recovery of oil, all of which require the use of non-Newtonian fluids. In light of the foregoing, this study presents a novel visualisation for analysing the effects of nonuniform heat generation/absorption, velocity slip, and viscous

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dissipation with heat transfer flow of viscoelastic fluid caused by a stretched surface embedded in a porous medium. Recently used the Runge-Kutta method taking into account the heat transfer effect to examine the coating of wires with a viscoelastic Oldroyd 8-constant fluid. Heat convection over a nonlinear sheet with a temperature-dependent viscosity was studied also performed an investigation of heat-driven MHD viscous flow. We have acquired series and analytical solutions, and we have explored the effects of new parameters using graphs. Looked at the effects of different factors of interest on a steady flow of a power-law fluid through an artery with a stenosis. Performed a comprehensive study of MHD flow and heat transfer via a viscoelastic fluid in the presence of a porous media in wire coating examination.

In this work, we use the semi-analytical method (HAM) with slip conditions to analyse the two-dimensional flow of a viscoelastic fluid with a nonuniform heat source along a permeable stretched sheet. Through the use of similarity variables, the modelled partial differential equations are transformed into ordinary differential equations, and HAM is used to generate solutions in a series. The impact of new parameters in the solution is described in depth using visual representations. The results are double-checked by comparing them to those found in a previous study.

DEFINITION OF THE ISSUE

In this part, we will examine the boundary layer flow of an incompressible viscoelastic fluid across a stretching sheet embedded in a porous material in two dimensions. The origin is a slit through which the sheet (shown in Figure 1) is pulled through the fluid medium.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu}{\rho}\frac{\partial^2 u}{\partial x^2}-\frac{\mu_e}{\rho k}u -\frac{k_0}{\rho}\left(u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{k}{\rho c_p}\left(\frac{\mu_e}{k}u^2+u\left(\frac{\partial u}{\partial x}\right)^2\right)+\frac{q'''}{\rho c_p}, \tag{3}$$

The slip condition's boundary conditions can be represented as

$$\begin{gathered} u=U+a\left(\frac{\partial u}{\partial y}-\frac{k_0}{\mu}\left(u\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^2 u}{\partial y^2}+2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}\right)\right),\\ v=-v_w,\ T_w=T_\infty+Ax^r \text{ at } y=0,\\ u\longrightarrow 0,\ T_w\longrightarrow T_\infty, \text{ as } y\longrightarrow\infty, \end{gathered} \tag{4}$$

In order to simplify the mathematical analysis of the problem, we introduce the following dimensionless coordinates, where an is the velocity slip factor:

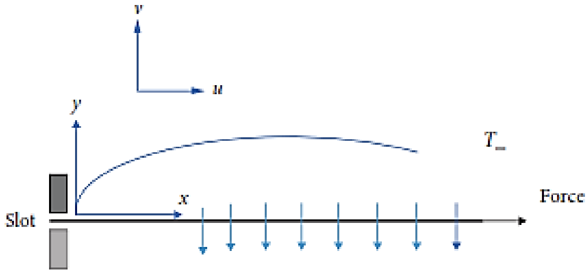

Figure 1: Representation of the physical state

$$\begin{gathered} \eta=y\sqrt{\frac{c}{\upsilon}},\\ u=cxf'(\eta),\\ v=-\sqrt{c\upsilon}f(\eta),\\ \theta(\eta)=\left(\frac{T-T_\infty}{T_w-T_\infty}\right), \end{gathered} \tag{5}$$

$$q'''=\left(\frac{kU}{\nu x}\right)\left[a^*(T-T_\infty)e^{-\eta}+b^*(T_w-T_\infty)\right]. \tag{6}$$

This means that the governing equations (1)-(3) for the boundary layer can be formulated in the dimensional shape that looks like this:

$$f'''-(f')^2+ff''-\beta f'+k\left((f'')^2-2f'f'''+ff''''\right)=0, \tag{7}$$

$$\frac{1}{\Pr}\theta''+f\theta'-2f'\theta+Ec\left(\beta(f')^2+(f'')^2\right)+\frac{1}{\Pr}\left(a^*e^{-\eta}+b^*\theta\right)=0, \tag{8}$$

In addition to the following boundary conditions:

$$f=f_w,\ f=1+\lambda\left[(1+3kf')f''+kf_wf'''\right],\quad \theta=1, \text{ at } \eta=\infty, \tag{9}$$

$$f'\longrightarrow 0,\ f''\longrightarrow 0,\ \theta\longrightarrow 0, \text{ at } \eta\longrightarrow\infty, \tag{10}$$

METHODS

HAM

We use the homotopy analysis technique using the following process to solve equations (7) and (8) subject to boundary conditions (9) and (10). As an auxiliary parameter, Z governs and limits the convergence of the solutions found by applying the main solution. This is how we pick our first set of guesses:

When choosing the initial approximations, we make sure they meet the boundary conditions in the following ways:

$$f_0(\eta) = s - 1 + e^{-\eta}, \quad \theta_0(\eta) = e^{-\eta}. \tag{11}$$

$$\Im_f(f) = f''', \quad \Im_\theta(\theta) = \theta''. \tag{12}$$

$$\Im_f\left(c_1 + c_2\eta + c_3\eta^2 + c_4 e^{-\eta}\right) = 0, \quad \Im_\theta\left(c_5 + c_6 e^{-\eta}\right) = 0, \tag{13}$$

To define the nonlinear operators, we use (7) and (8).

$$\begin{aligned}\aleph_f[f(\eta;p)] = {} & \frac{\partial^3 f(\eta;p)}{\partial\eta^3} - \left(\frac{\partial f(\eta;p)}{\partial\eta}\right)^2 \\ & + f(\eta;p)\frac{\partial^2 f(\eta;p)}{\partial^2\eta} - \beta\frac{\partial f(\eta;p)}{\partial\eta} \\ & + \kappa\left(\left(\frac{\partial^2 f(\eta;p)}{\partial^2\eta}\right)^2 - 2\frac{\partial f(\eta;p)}{\partial\eta}\frac{\partial^3 f(\eta;p)}{\partial\eta^3}\right. \\ & \left. + f(\eta;p)\frac{\partial^3 f(\eta;p)}{\partial\eta^3}\right) = 0.\end{aligned}$$

$$\begin{aligned}\aleph_\theta[f(\eta;p),\theta(\eta;p)] = {} & \frac{1}{\Pr}\frac{\partial^2\theta(\eta;p)}{\partial\eta^2} + \Pr f(\eta;p)\frac{\partial\theta(\eta;p)}{\partial\eta} \\ & - 2f(\eta;p)\frac{\partial\theta(\eta;p)}{\partial\eta} \\ & + E_c\left(\beta\left(\frac{\partial\theta(\eta;p)}{\partial\eta}\right)^2 + \left(\frac{\partial^2\theta(\eta;p)}{\partial\eta^2}\right)^2\right) \\ & + \frac{1}{\Pr}\left(a^* e^{-\eta} + b^*\theta\right) = 0.\end{aligned} \tag{14}$$

RESULTS AND DISCUSSION

Over a permeable stretched sheet embedded in a porous medium, a non-Newtonian viscoelastic fluid' has been studied, exhibiting non-uniform heat generation in two dimensions. The PDEs have been converted to ODEs using the similarity transformation. Applying HAM, an analytical answer was found. For the purpose of verifying the accuracy of our analytical solution, we have compared it to the research. Table 1 provides a summary and comparison of the data. This guarantees that our study is of the highest quality and consistent with previous research. Figures 4-13 show the computed findings.

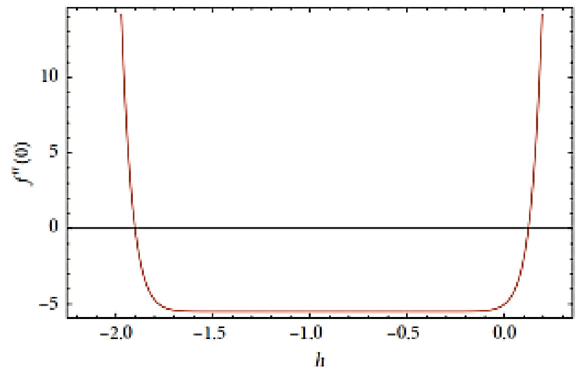

Figure 2: Field of Velocity h-curve

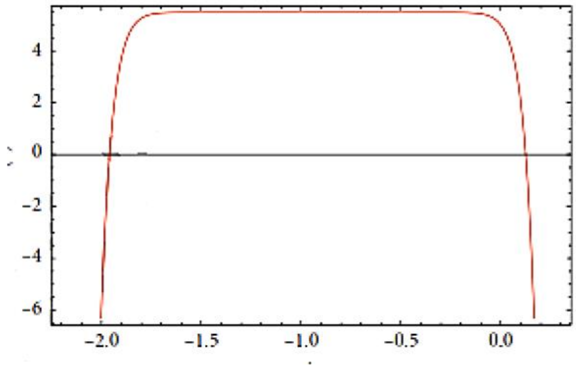

Figure 3: Field temperature h-curve

As seen in Figure 4, the fluid velocity drops for larger values of the porosity parameter. When beta is large, the dynamic viscosity is also large, indicating the presence of a porous medium with low permeability, as seen in Figure 5. This produces a resistance force to the fluid ow, which in turn decreases the velocity distribution and strengthens the boundary layer. In addition, this picture demonstrates how an increase in the porosity parameter results in a thicker thermal boundary layer and a thinner momentum boundary layer.

Figures 6 and 7 depict the impact of Eckert number E on velocity and temperature distributions. As can be seen in Figure 6, the momentum effect is diminished as the velocity curve dips with increasing Eckert number. Figure 7 also shows that as the Eckert number increases, the thermal boundary layer grows in thickness, but the temperature distribution improves.

Table 1: Evaluation of the current work in relation to the previously released one

	Rajagopal et al. [24]	Present work
0.0	0.98561340	0.98561423
1.0	0.27908819	0.27908734
2.0	0.09291179	0.09291328
3.0	0.03295374	0.03295452
4.0	0.01196183	0.01196265

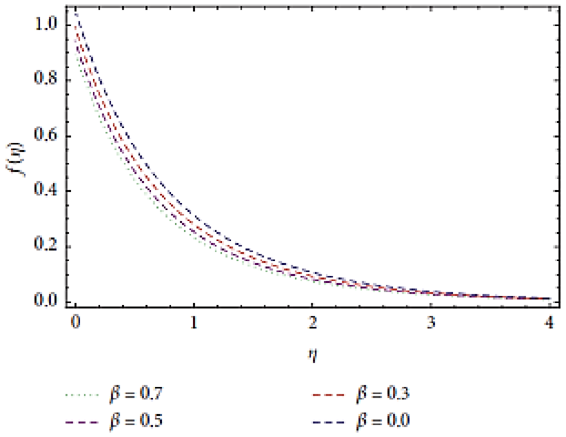

Figure 4: Effect of velocity profile for various values

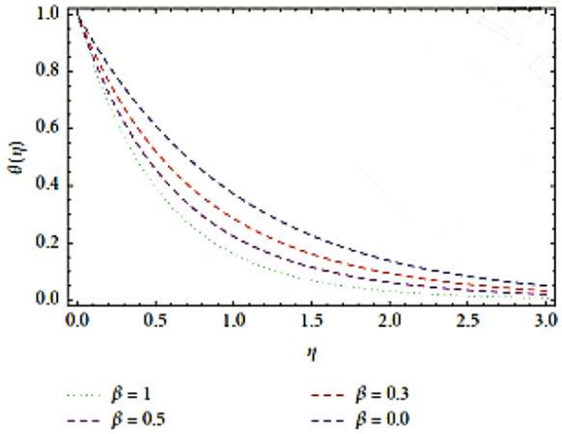

Figure 5: How the temperature distribution changes for various values of

The influence of the slip parameter against the similarity variable on the velocity and temperature profiles is displayed in Figure 8 and 9. It is found that as the slip velocity parameter is increased, the fluid's velocity slows and the temperature rises.

Figure 10 and 11 display the results of an analysis of the impact of the suction parameter fw on the fluid flow and temperature profile. The thickness of the boundary layer is significantly affected by the suction parameter, as demonstrated by the figures. As the suction parameter is increased, the boundary layer thickens, and the fluid flow and temperature distribution are both attenuated.

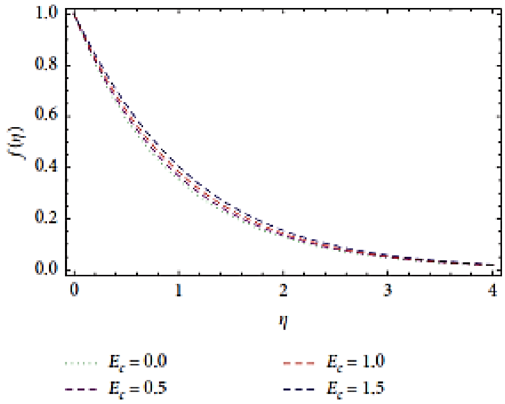

Figure 6: The influence of the velocity profile for different values of E_c, when $K = 0.1, f_w = 0.3, \lambda = 0.2, \Pr = 5.0, \beta = 0.4$, and $a^* = b^* = 0.2$.

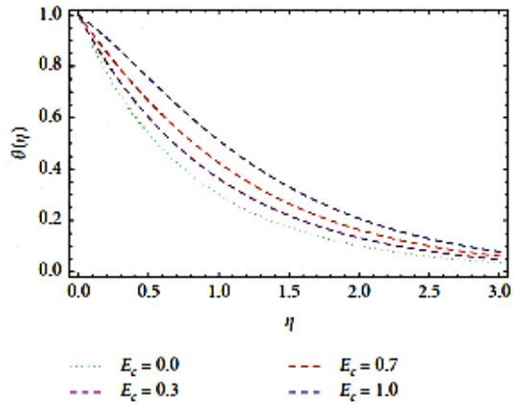

Figure 7: The influence of the temperature profile for different values of E_c, when $K = 0.1, f_w = 0.3, \lambda = 0.2, \Pr = 5.0, \beta = 0.4$, and $a^* = b^* = 0.2$.

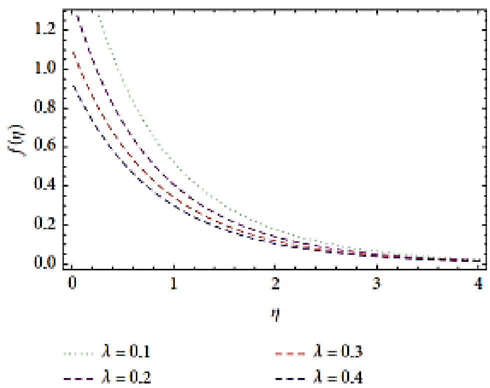

Figure 8: The influence of the velocity profile for different values of λ, when $K = 0.1, f_w = 0.3, \beta = 0.2, \Pr = 5.0, E_c = 0.4$, and $a^* = b^* = 0.2$.

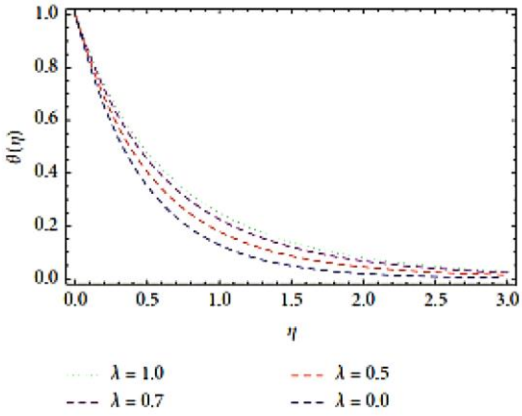

FIGURE 9: The influence of the temperature profile for different values of λ, when $K = 0.1$, $f_w = 0.3, \beta = 0.2, \Pr = 5.0, E_c = 0.4$, and $a^* = b^* = 0.2$.

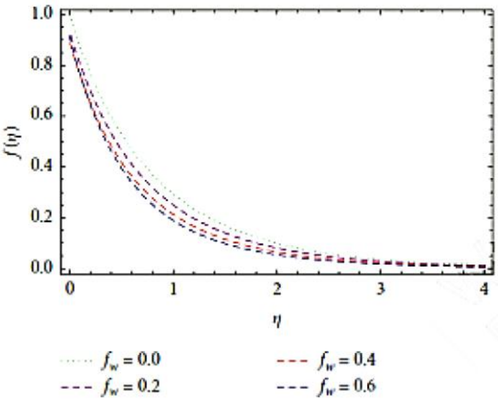

FIGURE 10: The influence of the velocity profile for different values of f_w, when $K = 0.1, \beta = 0.4, \Pr = 5.0, E_c = 0.4, \lambda = 0.2$, and $a^* = b^* = 0.2$.

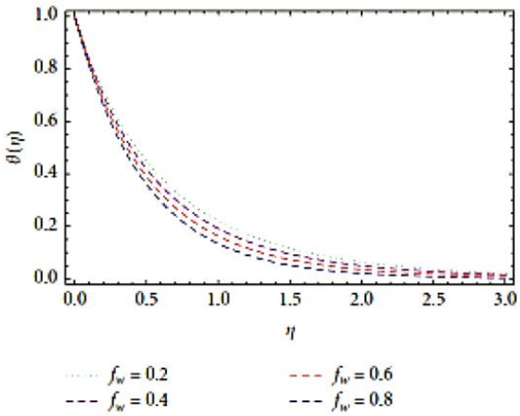

FIGURE 11: The influence of the temperature profile for different values of f_w, when $K = 0.1, \beta = 0.4, \Pr = 5.0, E_c = 0.4, \lambda = 0.2$, and $a^* = b^* = 0.2$.

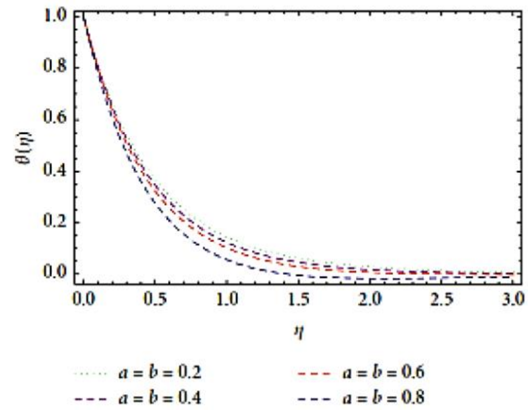

FIGURE 12: The influence of the velocity profile for different values of a^*, b^*, when $K = 0.1, \beta = 0.4, \Pr = 5.0, E_c = 0.4, \lambda = 0.2$, and $f_w = 0.3$.

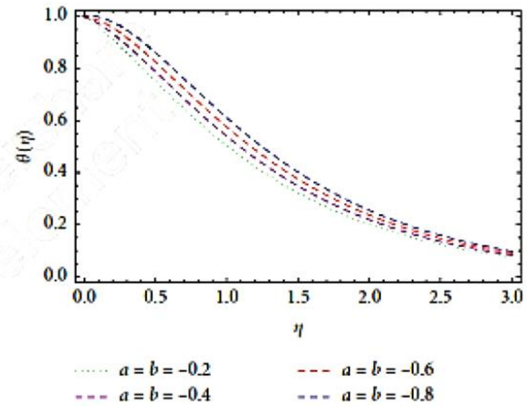

FIGURE 13: The influence of the temperature profile for different values of a^*, b^*, when $K = 0.1, \beta = 0.4, \Pr = 5.0, E_c = 0.4, \lambda = 0.2$, and $f_w = 0.3$.

CONCLUSION

The proposed model problem of heat transfer phenomena in a viscoelastic fluid through a stretching sheet surface embedded in a porous medium with viscous dissipation of internal heat generation/absorption and slip velocity is solved using a seminumerical methodology called homotopy analysis. We provide a graphical presentation of the method's convergence analysis. The impact of new parameters on the final answer has been extensively discussed. The boundary layer flow is shown to be thinner when the suction parameter is increased. In the same way that the suction parameter affects the thickness of the boundary layer flow, the porosity and slip parameters do as well. Further, when the Eckert number grows larger, so do the thermal boundary layer and the temperature distribution.

REFERENCES

Crane L.J., 2010. "Flow past a stretching plate," Zeitschrift f ur Angewandte Mathematik und Physik ZAMP, **21**(4): 645–647.

Vleggar J., 2007. “Laminar boundary- layer behavior on continuous accelerating surfaces,” Chemical Engineering Science, **32**(12): 1517–1525.

Dutta B.K., Roy P. and Gupta A.S., 2015. “Temperature field in flow over a stretching sheet with uniform heat flux,” International Communications in Heat and Mass Transfer, **12**(1): 89–94.

Pop I. and Na T.Y., 2008. “A note on MHD flow over a stretching permeable surface,” Mechanics Research Communications, **25**(3): 263–269.

Ali M.E., 2005. “On thermal boundary layer on a power-law stretched surface with suction or injection,” International Journal of Heat and Fluid Flow, **16**(4): 280–290.

Vajravelu K. and Rollins D., 2011. “Heat transfer in a viscoelastic fluid over a stretching sheet,” Journal of Mathematical Analysis and Applications, **158**(1): 241–255.

Andersson H.I., 1992. “MHD flow of a viscoelastic fluid past a stretching surface,” Acta Mechanica, **95**(1–4): 227–230.

Subhas A. and Veena P., 2008. “Visco-elastic fluid flow and heat transfer in a porous medium over a stretching sheet,” International Journal of Non-Linear Mechanics, **33**(3): 531–540.

Sanjayanand E. and Khan S.K., 2016. “On heat and mass transfer in a viscoelastic boundary layer flow over an exponentially stretching sheet,” International Journal of Thermal Sciences, **45**(8): 819–828.

Nandeppanavar M.M., Abel M.S. and Vajravelu K., 2018. “Flow and heat transfer characteristics of a viscoelastic fluid in a porous medium over an impermeable stretching sheet with viscous dissipation,” International Journal of Heat and Mass Transfer, **53**(21-22): 4707–4713.

Navier C.L.M.H., 2003. “M emoire sur les lois du mouvement des fluids,” Memoires de l’Acaddmie des Sciences de l’lnstitut de France, **6**: 389–416.

Thompson P.A. and Troian S.M., 2007. “A general boundary condition for liquid flow at solid surfaces,” Nature, **389**(6649): 360–362.