

K-BEST SPHERE DECODER ALGORITHM FOR SPATIAL MULTIPLEXING MIMO SYSTEMS

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ABSTRACT

To achieve high data rates, reliable data rate transmission and near capacity performance of next generation wireless communication systems iterative processing has been widely considered. However, such an efficient receiver design has been made significant challenge. In this paper, MIMO detection techniques have been investigated which achieves reliable data transmission with less computation complexity. We studied and review the MIMO receiver algorithms such as Sphere decoding (SD), K-best decoding and interference cancellation. In order to reduce computational complexity without significant performance degradation low complexity K-best (LC-K best) based SD is evaluated.

KEYWORDS: Iterative Detectors; K-Best SD; LC-K-Best SD; MIMO; Sphere Decoder.

Wireless Communication systems have known a big evolution over the last decade. Their developments have been driven by the increase of human demand to get information rate with better quality of service (QOS) in shortest possible time and at highest speed. However, many challenges arise and are directly related to the limited transmission power, frequency spectrum allocation and channel propagation issues as time and frequency fading.

MIMO technology employs multiple antennas both at transmitter and receiver to achieve high diversity through space time coding and high data rate through spatial multiplexing(SM) without the need of additional spectrum and transmit power.

Further the techniques for the implementation of iterative processing in MIMO OFDM systems has been evolving to achieve a trade off between BER performance and complexity

The aim of this paper is to address the various challenges involved by the iterative receiver combining MIMO detection. Therefore, an advanced receiver must be developed at algorithmic and architectural levels to achieve near optimal performance with tolerable computational complexity. The receiver must also satisfy high throughput, low latency and low power consumption requirements for wireless communication systems.

Novel Contribution of this paper can be summarised as

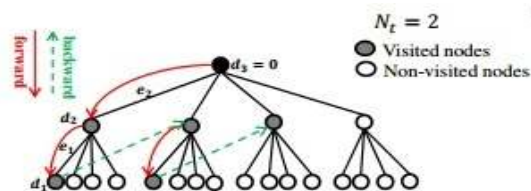
In literature various detection algorithms such as SISO detection algorithms for iterative processing of MIMO receivers has been studied.

For the benefits of low computational complexity and latency without significant performance degradation of MIMO system, the LC-K-best has been studied and evaluated in this paper.

Complexity and performance analysis of LC-K best decoder has been compared with existing MIMO detection algorithms with different modulation schemes, channel models and different configuration. Therefore, based on simulation results LC-K-best decoder algorithm achieves optimal performance with low complexity compared to conventional techniques.

TREE-SEARCH BASED DETECTION

The detection algorithms problem can be eliminated with help of tree-search based algorithms [Wubben et.al., 2001 & Damen et.al., 2003]. Large number of tree search based detection algorithms has been reported in the literature which archived near ML performance with low computational complexity. Some of which are shown in Figure 1.

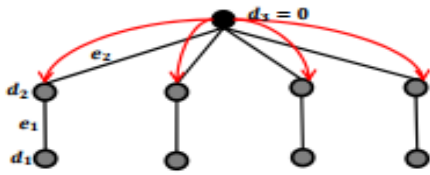


(a) tree search based on Depth first SD



(b) tree search based on Breadth first K-Best decoder

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(c) Breadth first search Fixed SD

Figure 1: different Tree-search strategies

Depth First Search SD

This algorithm is used to solve the detection problem and achieve near optimal performance with polynomial average computational complexity for a large range of SNR [Hassibi and Vikalo, 2001].

The SD was originally defined for the computation of minimal length lattice vectors [Pohst, 1981]. Further improved methods has been introduced for calculating the short lattice vectors [Fincke and Pohst, 1985], it was then used for ML estimation. Viterbo and Biglieri applied the Fincke-Pohst (FP) algorithm to lattice decoding [Viterbo and Biglieri, 1993]. Schnorr and Euchner [Schnorr and Euchner, 1994] proposed a refinement to the FP algorithm. Viterbo and Boutros used lattice code decoding in fading channels [Viterbo and Boutros] and Damen used lattice code decoder for STC. The basic idea of sphere detection is to limit the search space of optimal ML solution to a hyper sphere of radius r_s around the received vector as shown in Figure 2.

Therefore, only lattice points that lie inside the hyper sphere are tested instead of testing all the hypotheses of the transmitted signal, reducing the computational complexity

$$\hat{s}_{SD} = \arg \min_{s \in 2^{QN_t}} \left\{ \|y - Hs\|^2 \leq r_s^2 \right\} \quad (1)$$

The channel matrix H, as we have decomposed into two matrix Q and R, $H = QR$. Therefore by using the QRD, the detection problem is equivalent to

$$\hat{s}_{SD} = \arg \min_{s \in 2^{QN_t}} \left\{ \|\tilde{y} - Rs\|^2 \leq r_s^2 \right\} \quad (2)$$

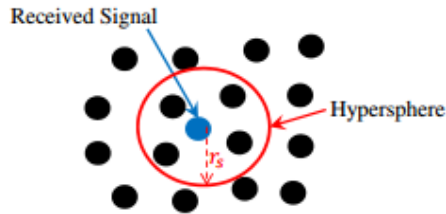


Figure 2: Sphere decoder principle.

Where R is the triangular nature, the Euclidean distance can be defined as $d_1 = \|\tilde{y} - Rs\|^2$ which can be evaluate the accumulated partial Euclidean distance (PED) d_i with $d_{N_t+1} = 0$ recursively therefore, the PED is given as

$$d_i = d_{i+1} + \left| \tilde{y}_i - \sum_{j=i}^{N_t} R_{i,j} s_j \right|^2 = d_{i+1} + |e_i|^2 \quad (3)$$

This process can be illustrated in Figure 3 based on tree search with $N_t + 1$ node, where i th level representing as i th transmit antenna.

Searching algorithm start with root node or level with first child node at N_t level, which represents the N_t th antenna transmitted symbol. Then partial Euclidean distance (PED) is derived. If the PED d_{N_t} represent the sphere radius r_s , then the searching process continuous up to $N_t - 1$ level and steps down process at i th level until evaluate a valid leaf node at first level. The first found point with the depth first search SD is the Babai point (BP), which corresponds to QRD-based solution [Agrell et.al., 2002 & Damen et.al., 2003].

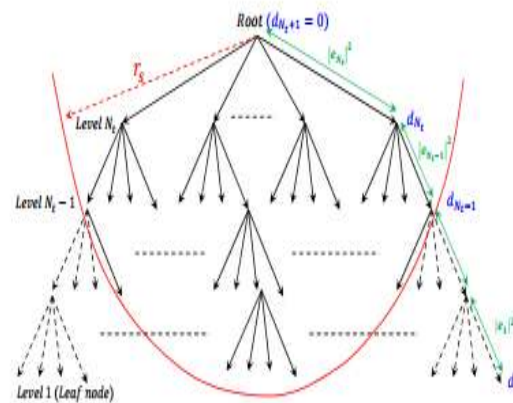


Figure 3: Tree-search representation of MIMO detection

The tree-search can be also represented using a linear ZF filter with a Cholesky decomposition of Gram matrix ($H^T H$) instead of QR decomposition. We note that both approaches are equivalent and give the same path metrics and candidates. Through our work, QR decomposition is used to describe the tree-search problem.

Enumeration Strategies

Enumeration strategy refers to the order in which the children of a node are tested. Two enumeration strategies can be used: Fincke-Pohst (FP) [Fincke and Pohst, 1985 & Viterbo and Biglieri, 1993] and Schnorr- Euchner (SE) [Schnorr and Euchner, 1994] as represented in Figure 4.



Figure 4: Enumeration strategies: (a) FP and (b) SE.

FP enumeration tries to find the shortest lattice vector by traveling the tree in forward and backward directions without any ordering. SE enumeration is improvement of FP enumeration by ordering ascending order with respect to their Euclidean distance where closer hypothesis will be tested first [Schnorr and Euchner, 1994].

A low complexity SE decoding algorithm was therefore proposed for QAM modulation in [Guo and Nilsson, 2004]. Figure 4 illustrates these two enumeration strategies, where the numbers represent the order in which the hypotheses are tested.

The enumeration strategy has a major impact on the complexity of the search. Obviously, the use of SE strategy leads to reduction in the computational complexity. This is due to the fact that most probable hypothesis is first tested which reduces the number of visited nodes during the search, and avoids the computation of branch metrics for paths which will be subsequently discarded [Viterbo and Biglieri, 1993 & Guo and Nilsson, 2004].

Radius Choice and Tree Pruning Criteria

One important challenge of the sphere decoder is the choice of an initial value of the search radius r_s . Clearly, if an r is chosen too large, the number of visited nodes may be very high and then the complexity will be increased in an exponential manner, whereas if an r is chosen too small, there may be no nodes inside the hyper sphere.

A simple approach consists in increasing radius search (IRS) as proposed in [Viterbo and Boutros & Hassibi and Vikalo, 2005].

In this case, the radius is first initialized to a fixed value r_0 . If no candidate is found, the search must be repeated using a larger radius ($r_1 > r_0$) which dramatically increases the detector latency. In [Zhang and Fossorier, 2005] an improved increasing radius search algorithm was proposed. This algorithm exploits the most promising candidates in the incomplete tree when the search fails in order to avoid the redundant computation of branch metrics for the starting search.

Therefore, the use of a fixed sphere radius is not efficient for practical systems [Hassibi and Vikalo, 2001]. The efficient solution for the initial radius choice is to use an adaptive approach. It consists in initializing the radius with an infinite value and updating it whenever a valid leaf node reached [Agrell et.al., 2002].

In the tree-search, when the partial Euclidean distance of a given node exceeds the search radius, this node is pruned. This algorithm uses a radius with a pruning probability for each layer. In a statistical tree pruning approach was proposed. This method uses a probabilistic noise constraint to tighten the necessary condition on each layer. Figure 2.4a shows an example of SD for $N_t = 2$, where solid lines and dash lines represent the forward and backward search in the tree, respectively.

It has been shown in [Hassibi and Vikalo, 2001 & Hassibi and Vikalo, 2005] that the sphere decoder achieves quasi-ML performance with polynomial average computational complexity in terms of the number of transmits antennas. However, the worst case presents an exponential complexity [Jalden and Ottersten, 2005]. From an implementation point of view, the SD has two main drawbacks. Firstly, its variable complexity which depends on the noise level and the channel conditions making it unsuitable for constant rate applications. Secondly, the sequential nature of the tree-search limits the performance and the level of parallelism in hardware implementation.

Although SD offers significant reduction in complexity compared to ML decoder, it still requires considerable computational complexity. In order to reduce the computational complexity of SD and to obtain a constant throughput, other implementation strategies and sub-optimal algorithms have been developed such as K-Best decoder [Wong et.al., 2002]

and fixed sphere decoder [Barbero and Thompson, 2006].

K-Best Decoder

The K-Best algorithm [Wong et.al., 2002] is based on breadth-first search in which the tree is traversed only in the forward direction. This approach commonly denoted as M-algorithm constructs the tree layer by layer retaining only a fixed number K of paths with best metrics at each detection layer. In Figure 2.4c illustrated that the tree-search with $N_t = 2$. The algorithm starts by extending the root node to all possible candidates and then sorting new path based on their metrics and keep the best possible K paths with smallest metrics for next detection layer. The K-best detection algorithms summarized as

1. for layer $i = N_t$ to 1 Do
2. Extend each survivor path to all $\sqrt{(2^Q)}$ possible paths
3. Update the PED metric for each path
4. Sort the paths according to their PED metrics
5. Select K best paths and updates the path history accordingly
6. If layer = 1, stop the algorithm else go to step 2.

K-Best algorithm is able to achieve near optimal performance with a fixed complexity and suitable level for parallel implementation. This fixed complexity depends on the number K of retained candidates, on the size of modulation and on the number of transmit antenna, where the number of visited nodes in the tree is equal to $2^Q + (N_t - 1)K2^Q$.

However, K-Best algorithm does not consider the noise variance and channel conditions. In addition, two main limitation of K-Best detection algorithm has the expansion and the sorting operations.

Best algorithm expands each K retained paths to its 2^Q possible children at each level. Thus, a high complexity is required to enumerate the children nodes especially in the case of higher order modulation and higher number of survival paths. For this reason, several enumeration schemes have been proposed in complex domain to avoid the full expansion such as phase shift keying (PSK) enumeration, relaxed K-Best enumeration [Chen et.al., 2007] and on demand expansion [Shabany and Gulak, 2008 & Wiesel et.al., 2003]. Meanwhile, in real signal model, the enumeration can be done through a slicing operation to

the nearest constellation point or simply through the use of a LUT [Wiesel et.al., 2003]. Recognizing the low efficiency of M-algorithm with high-order modulation, multi-level enumeration methods have been proposed in [Jong and Willink, 2005]. This approach partitions the constellation into different sub-segments such that each layer is effectively divided into sub-layers.

Furthermore, the algorithm requires computing and sort $2^Q K$ path metrics at each level of which $K(2^Q - 1)$ belonging to paths are pruned from the tree. This sorting algorithm is very time consuming.

Moreover, the algorithm is prone to error propagation especially for low values of K. One way of tracking this problem is to use an adaptive value of K as a function of the tree depth [Wenk et.al., 2006 & Shabany and Gulak, 2012]. A large value of K is used for the first layer which is then reduced when detecting the last layers.

The first implementation of K-Best decoder is [Wong et.al., 2002] for 4x4 16-QAM MIMO system. Different VLSI implementations have been subsequently proposed in the literature to improve the algorithm performance [Wenk et.al., 2006 & Liu et.al., 2010].

Fixed Complexity Sphere Decoder(FSD)

The fixed sphere decoder (FSD) is another sub-optimal MIMO detection scheme further reduces the complexity of K-Best decoder [Barbero and Thompson, 2006, & Barbero and Thompson, 2006]. The performance of FSD is based on two stages of tree-search as illustrated in Figure 1(d).

- Full expansion: A full expansion is performed at first p top levels, where all possible candidates are retained to the following detection levels.
- Single expansion: Perform a single search process up to remaining levels ($N_t - p$), where only one candidate per node having lowest metric is considered for next layers.

In FSD detection process the columns of H must be ordered as the first p levels then the signal has the largest post-processing noise amplification. Mean while remaining ($N_t - p$) levels sorted based on their reliability with least amplification noise has been detected first.

The conventional FSD has a fixed complexity however it does not take into consideration the noise and channel conditions. In [Xiong et.al., 2009], a

simplified version of the FSD has been proposed by introducing the path selection of the remaining levels. FSD algorithm can be highly parallelized and fully pipelined. Several implementations of FSD have been reported in [Barbero and Thompson, 2006 & Khairy et.al., 2009].

PERFORMANCE SIMILARION RESULTRS AND DISCUSSION

Simulation Parameter

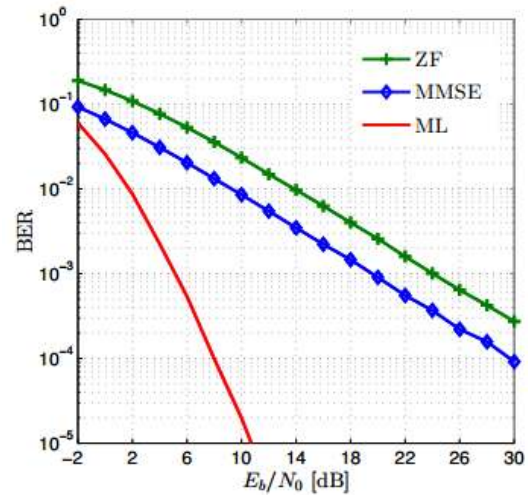
In this section, we compare the uncoded BER performance of the most prominent hard-decision detection algorithms. The simulation results performed on 4×4 MIMO systems with QAM under Rayleigh fading channel, randomly generated Rayleigh fading channel with zero mean and unit noise variance. The simulation parameters are shown in Table 1.

The performance of spatial multiplexing (SM) MIMO system can be defined in terms of bit error rate (BER), therefore energy per bit information bit E_b/N_0 is defined as

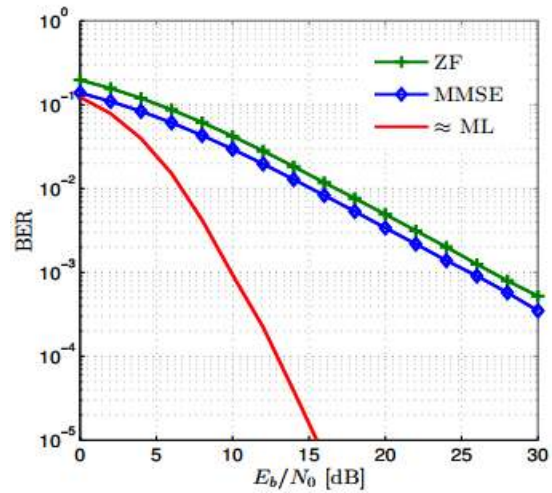
$$\frac{E_b}{N_o} = \frac{E_s}{N_o} + 10 \log_{10} \frac{1}{Q N_t}$$

BER Performance

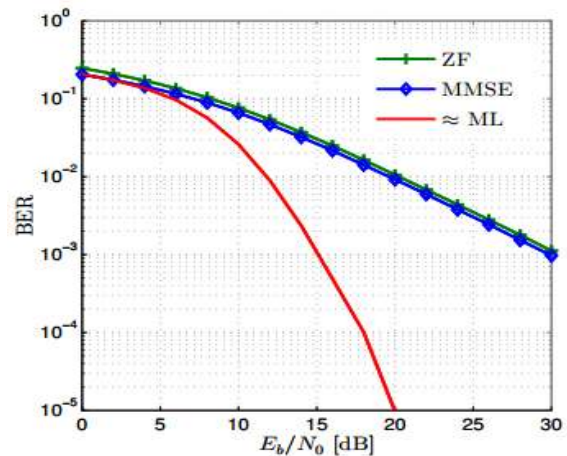
Figure 5 shows that the performance comparison of linear detectors in 4×4 MIMO system with 4-QAM, 16-QAM and 64-QAM modulation schemes. We show that the ML detector is obviously the optimum one and achieves a full diversity order equal to $N_r = 4$. The SD is used the ML solution in the case of 16-QAM and 64-QAM. MMSE detector shows better BER performance than ZF but both shows a same diversity order equal to one. Moreover, the gain of MMSE over ZF is reduced with a high-order modulation (64-QAM). In general, linear detectors present significant performance loss compared to the ML detector.



(a) 4-QAM



(b) 16-QAM



(c) 64-QAM

Figure 5: BER Performance of a 4×4 uncoded MIMO system of linear detectors using 3 constellations: (a) 4-QAM, (b) 16-QAM, (c) 64-QAM.

Table 1: Simulation Parameters

MIMO System	4 × 4 Spatial multiplexing
Modulation 2 ^Q -QAM	4-QAM,16-QAM & 64-QAM
Channel type	Rayleigh fading
MIMO Detector	ZF, MMSE SIC-ZF, SIC-MMSE, OSIC-ZF OSIC-MMSE, SQRD SD, K-Best, FSD
Channel decoder	R _c = 1 uncoded

In Figure 6 illustrated that BER performance of SIC detectors. SIC detectors are either based on V-BLAST algorithm with or without ordering (SIC or OSIC), or based on SQRD algorithm (SQRD). The SIC detectors achieve better performance compared to linear detectors as shown in Figure 5, but still show significant performance degradation in the high SNR compared to ML detector. We also shows that the ordering of symbol used to improvement of BER performance for both SIC-ZF and SIC-MMSE algorithms.

This improvement is better in the case of SIC-MMSE which indicates less error propagation compared to SIC-ZF. Furthermore, it is interesting to note that with the increase of modulation order, the improvement is reduced. Moreover, BER performance of SQRD and SQRD-MMSE based detectors is depicted in the case of 4-QAM and 64-QAM. Obviously SQRD-MMSE has better performance than SQRD. As SQRD-MMSE does not assure the optimal order, a performance gap between SQRD-MMSE and OSIC is observed.

Despite of layer ordering, none of these algorithms achieves full diversity order. Their diversity order lies between $N_r - N_t + 1 = 1$ and $N_r = 4$ and converges approximately to one for high SNR. SQRD-based detection has much lower computational complexity than the V-BLAST algorithm with a tolerable degradation in BER performance especially in case of high-order modulation.

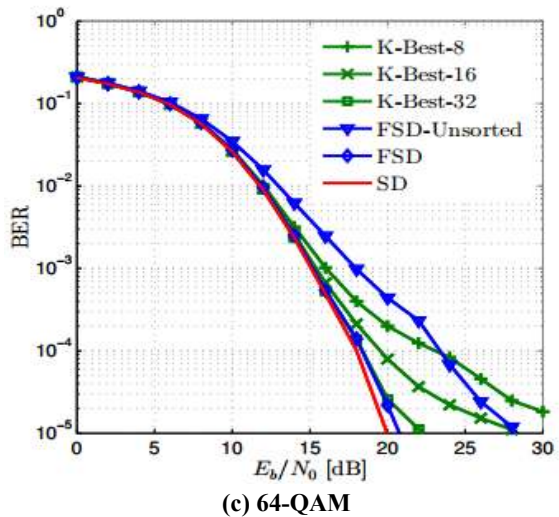
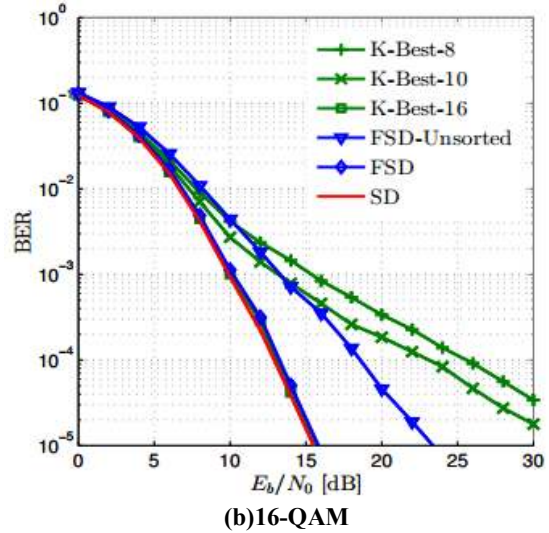
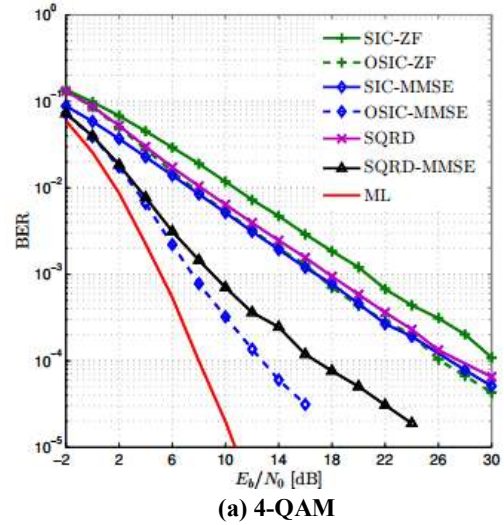


Figure 6: BER Performance of a 4×4 un-coded MIMO system of SIC detectors using 3 constellations: (a) 4-QAM, (b) 16-QAM, (c) 64-QAM.

CONCLUSION

In this paper, we have investigated different MIMO detection algorithms including linear detection, interference cancellation and tree-search based detection. Their associated advantages and drawbacks have been presented and discussed. We have finally compared their performance with different modulations. Until now, only hard-decision MIMO detection is considered in which the detector delivers a hard estimates of the transmitted symbols. However, in the case of channel coding, the performance of the system can be further improved by using soft-decision values. This soft information can be iteratively exchanged in order to achieve near capacity.

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