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# SIMULTANEOUS HONG-MANDEL'S HIGHER-ORDER SQUEEZING OF BOTH QUADRATURE COMPONENTS IN SUPERPOSITION OF TWO COHERENT STATES

**PANKAJ KUMAR**[1]

Department of Physics, Bhavan's Mehta Mahavidyalaya (V. S. Mehta College of Science), Bharwari, Kaushambi, U.P, India

## ABSTRACT

**We study simultaneous Hong-Mandel's higher-order squeezing of the two quadrature components by considering Hermitian operator, $X_\theta = X_1 \cos\theta + X_2 \sin\theta$, in the most general superposition state $|\psi\rangle = Z_1|\alpha\rangle + Z_2|\beta\rangle$, of two coherent states $|\alpha\rangle$ and $|\beta\rangle$. Here, Hermitian operators $X_{1,2}$ are defined by $X_1 + iX_2 = a$, a is the annihilation operator, is an arbitrary angle, $Z_1$, $Z_2$, α and β are arbitrary complex numbers and the only restriction on these is the normalization condition of the superposed coherent states. We conclude that simultaneous Hong-Mandel's higher-order squeezing of both quadrature components with equal minimum higher-order fluctuations occurs for an infinite number of combinations of parameters $Z_1$, $Z_2$, α, β and θ. Variations of higher-order fluctuations with intensity parameter have also been discussed.**

**KEYWORDS :** Non-Classical States, Squeezing, Hong-Mandel's Squeezing, Displacement Operator, Phase Shifting Operator

States of light, whose properties cannot be explained on the basis of classical theory, are called non-classical states (Walls, 1983; Loudon, 1987; Dodonov, 2002). The non-classical nature of a quantum state can be manifested in different ways like antibunching, sub-Poissonian photon statistics and various kinds of squeezing etc. Earlier study of such non-classical effects was largely in academic interest but now their applications in quantum information theory such as communication, quantum teleportation, dense coding and quantum cryptography are well realized.

Squeezing, a well-known non-classical effect, has been generalized to case of several variables (Hong et. al., 1985; Hillery, 1987; Zhang et. al., 1990). Hong and Mandel (Hong et. al., 1985) introduced the concept of higher-order squeezing by considering the $2n^{th}$ order moments of the quadrature component and defined a state to be $2n^{th}$ order squeezed if the expectation value of the $2n^{th}$ power of the difference between a field quadrature and its average value is less than what it would be in a coherent state. According to Hong-Mandel's definition, a state $|\psi\rangle$ is said to be $2n^{th}$-order squeezed for the operator,

$$X_\theta = X_1 \cos\theta + X_2 \sin\theta \tag{1}$$

if the $2n^{th}$-order moment of $X_\theta$,

$$\langle\psi|(\Delta X_\theta)^{2n}|\psi\rangle < 2^{-2n}(2n-1)!!, \tag{2}$$

where Hermitian operators $X_{1,2}$ are defined by $X_1 + iX_2 = a$, a is the annihilation operator, θ is an arbitrary angle, $\Delta X_\theta = X_\theta - \langle\psi|X_\theta|\psi\rangle$ and (2n-1)!! is product of factors, starting with (2n-1) and decreasing in steps of 2 and ending at 1. Note that the right hand side in inequality (Eq. (2)) is the value of left hand side for coherent state. Hong-Mandel's higher-order squeezing is quite distinct from ordinary squeezing because such squeezing does not require that the uncertainty product be a minimum and therefore both quadrature components of the field can have higher-order squeezing simultaneously (Lynch, 1986; Lynch, 1994; Kumar et. al., 2013). In other words, states exist for which product of higher-order fluctuations of both quadrarture takes a value less than that for a coherent state.

A coherent state does not exhibit any non-classical effect but superposition of coherent states exhibit several non-classical effects (Prakash et. al., 2008; 2011). Jackiw state, a superposition of coherent state $|\alpha\rangle$ and vacuum state $|0\rangle$, exhibits (Lynch, 1986) fourth-order squeezing in the two quadratures. Lynch studied (Lynch, 1994) simultaneous fourth-order squeezing of both quadrature components in orthogonal even coherent state, a superposition of, $|\alpha\rangle, |-\alpha\rangle, |i\alpha\rangle$ and $|-i\alpha\rangle$ and reported simultaneous fourth-order squeezing of both quadratures with equal minimum value 0.1746 of fourth-order fluctuations. Recently we generalized (Kumar et. al., 2013) the results of Lynch (Lynch, 1994) for higher-order squeezing. Prakash et. al. (Prakash et. al., 2007) studied simultaneous fourth-order squeezing in superposition of two coherent states. In the present paper we study simultaneous occurrence of Hong-Mandel's $2n^{th}$-order squeezing of both quadrature components in the most general superposition state,

[1]Corresponding author

$$|\psi\rangle = Z_1|\alpha\rangle + Z_2|\beta\rangle \qquad (3)$$

of two coherent states and . Here, complex numbers $Z_1$, $Z_2$, $\alpha$ and $\beta$ are all-arbitrary and the only restriction on these is normalization condition of the superposed state .

**HIGHER-ORDER MOMENTS OF $X_\Theta$ IN SUPEPROSED COHERENT STATES $|\psi\rangle$**

A single mode coherent state $|\alpha\rangle$ defined by a $|\alpha\rangle = \alpha|\alpha\rangle$ can be written as

$$|\alpha\rangle = \exp(-\frac{1}{2}|\alpha|^2)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle = D(\alpha)|0\rangle \qquad (4)$$

where $|n\rangle$ the occupation is number and $D(\alpha) = \exp(\alpha a^+ - \alpha^* a)$ is the displacement operator. Using the relation, $D^+(\alpha)\, a\, D(\alpha) = a+\alpha$, we have

$$\langle\psi'|(\Delta X_{\theta,|\psi'\rangle})^{2n}|\psi'\rangle = \langle\psi|(\Delta X_{\theta,|\psi'\rangle})^{2n}|\psi\rangle \qquad (5)$$

where, $\Delta X_{\theta.|\psi\rangle} = X_\theta - \langle\psi|X_\theta|\psi\rangle \equiv D(\alpha)|\psi\rangle$ ,

and $\Delta X_{\theta,|\psi\rangle} = X_\theta - \langle\psi'|X_\theta|\psi'\rangle$ From Eq. (5)

we conclude that the Hong-Mandel's $2n^{th}$-order squeezing in any state $|\psi\rangle$ is not affected by operation of the displacement operator. This observation and relation,

$$D(\alpha)\, D(\beta) = \exp[\tfrac{1}{2}(\alpha\beta^* - \beta\alpha^*)]\, D(\alpha+\beta), \qquad (6)$$

suggests that we can simplify the problem by writing the superposed coherent state $|\psi\rangle$ as

$$|\psi\rangle = D(\beta)|\psi_1\rangle; |\psi_1\rangle = K[|xe^{i\phi}\rangle + re^{i\xi}|0\rangle], \qquad (7)$$

where, $re^{i\xi} = \frac{Z_2}{Z_1}\exp[\frac{1}{2}(\alpha^*\beta - \alpha\beta^*)]$ and $xe^{i\phi} = (\alpha - \beta)$ Since $e^{i\theta N}ae^{-i\theta N} = ae^{-i\theta N} = ae^{-i\theta}; N = a^+a$, we can further write the state $|\psi\rangle$ as

$$|\psi_1\rangle = e^{i\phi N}|\psi_2\rangle; \quad |\psi_2\rangle = K[|x\rangle + re^{i\xi}|0\rangle] \qquad (8)$$

Now since

$$\langle\psi|(\Delta X_\theta)^{2n}|\psi\rangle = \langle\psi_1|(\Delta X_{\theta,|\psi_1\rangle})^{2n}|\psi_1\rangle, \qquad (9)$$

and also,

$$\langle\psi_1|(\Delta X_{\theta,|\psi_1\rangle})^{2n}|\psi_1\rangle = \langle\psi_2|(\Delta X_{\delta,|\psi_2\rangle})^{2n}|\psi_2\rangle \qquad (10)$$

$\delta = (\theta - \phi)$, therefore for studying maximum simultaneous Hong-Mandel's $2n^{th}$-order squeezing in both quadrature components with equal $2n^{th}$-order fluctuations in the superposed state $|\psi\rangle$, we minimize $S^{(2n)} \equiv \langle\psi_2|(\Delta X_{\delta,\psi_2\rangle})^{2n}|\psi_2\rangle$ with parameters $\delta$, x, r and $\phi$. Now we have

$$\langle\psi_2|(ae^{-i\delta})^n|\psi_2\rangle = K^2x^n[e^{-in\delta} + re^{-\frac{x^2}{2}}e^{-i(0+n\delta)}] \qquad (11)$$

and

$$\langle\psi_2|(a^+e^{i\delta})^m(ae^{-i\delta})^n|\psi_2\rangle = K^2x^{(m+n)}e^{-i(n-m)\delta} \qquad (12)$$

It is easier to study higher-order moment in the state $|\psi_2\rangle$ that to study higher-order moment in the state $|\psi\rangle$. For examples, we calculate fourth-order moment and sixth-order moment of $X_\delta$ in the state $|\psi_2\rangle$ and study the simultaneous occurrence of these squeezing effects in the superposed coherent state $|\psi\rangle$. We finally get fourth-order moment and sixth-order moment of $X_\delta$ in the superposed state $|\psi_2\rangle$ respectively,

$$\langle(\Delta X_\delta)^4\rangle = \langle:(\Delta X_\delta)^4:\rangle + \frac{3}{2}\langle:(\Delta X_\delta)^2:\rangle + \frac{3}{16} \qquad (13)$$

$$\langle(\Delta X_\delta)^6\rangle = \langle:(\Delta X_\delta)^6:\rangle + \frac{15}{4}\langle:(\Delta X_\delta)^4:\rangle + \frac{45}{16}\langle:(\Delta X_\delta)^2:\rangle + \frac{15}{64} \qquad (14)$$

Here for any operator X,: X : is its normal form, and

$$\langle:X_\delta^2:\rangle = \frac{1}{2}[\mathrm{Re}\{\langle a^2\rangle e^{2i\delta}\} + \langle a^+a\rangle]$$

$$\langle:X_\delta^3:\rangle = \frac{1}{4}[\mathrm{Re}\{\langle a^3\rangle e^{3i\delta}\} + 3\,\mathrm{Re}\{\langle a^+a^2\rangle e^{i\delta}\}] \qquad (15)$$

$$\langle:X_\delta^4:\rangle = \frac{1}{8}[\mathrm{Re}\{\langle a^4\rangle e^{-4i\delta}\} + 4\,\mathrm{Re}\{\langle a^+a^3\rangle e^{-2i\delta}\} + 3\langle a^{+2}a^2\rangle], \qquad (16)$$

$$\langle:X_\delta^5:\rangle = \frac{1}{16}[\mathrm{Re}\{\langle a^5\rangle e^{-5i\delta}\} + 5\,\mathrm{Re}\{\langle a^+a^4\rangle e^{-3i\delta}\} + 10\,\mathrm{Re}\{\langle a^{+2}a^3\rangle e^{-i\delta}\}], \qquad (17)$$

$$\langle:X_\delta^6:\rangle = \frac{1}{32}[\mathrm{Re}\{\langle a^6\rangle e^{-6i\delta}\} + 15\,\mathrm{Re}\{\langle a^{+2}a^4\rangle e^{-2i\delta}\} + 6\,\mathrm{Re}\{\langle a^+a^5\rangle e^{-4i\delta}\} + 10\langle a^{+3}a^3\rangle] \qquad (18)$$

$$\langle:(\Delta X_\delta)^2:\rangle = \langle:X_\delta^2:\rangle - \langle:X_\delta:\rangle^2 \qquad (19)$$

$$\langle :(\Delta X_\delta)^4: \rangle = \langle :X_\delta^4: \rangle + 6\langle :X_\delta^2: \rangle \langle :X_\delta: \rangle^2 - 3\langle :X_\delta^4: \rangle - 4\langle :X_\delta^3: \rangle \langle :X_\delta: \rangle \quad (20)$$

$$\langle :(\Delta X_\delta)^6: \rangle = \langle :X_\delta^6: \rangle - 6\langle :X_\delta^5: \rangle \langle :X_\delta: \rangle + 15\langle :X_\delta^4: \rangle \langle :X_\delta: \rangle^2 + 15\langle :X_\delta^2: \rangle \langle :X_\delta: \rangle^4$$

and $$-20\langle :X_\delta^3: \rangle \langle :X_\delta: \rangle^3 - 5\langle :X_\delta: \rangle^6 \quad (21)$$

## RESULTS

Using computer programming, we get minimum values 0.1840 of $S^{(4)} \equiv \langle \psi_2 | (\Delta X_{\delta,\psi_2})^4 | \psi_2 \rangle$ and $S^{(6)} \equiv \langle \psi_2 | (\Delta X_{\delta|\psi_2\rangle})^6 | \psi_2 \rangle$ 0.2197 of with $\delta = \pm \pi / 4$ at x = 1.55, ξ=0 and r = 0.038. Therefore, we finally conclude in terms of the parameters Z1, Z2, α, β and θ considered originally, that maximum simultaneous Hong-Mandel's fourth-order squeezing and Hong-Mandel's sixth-order squeezing in both quadrature components in the state $|\psi\rangle$ occurs with equal minimum value 0.1840 and 0.2197 respectively for infinite number of combinations with $\alpha - \beta = 1.55 \exp[i(\theta \pm \frac{\pi}{4})]$, $\frac{Z_2}{Z_1} = 0.038 \exp[\frac{1}{2}(\alpha\beta^* - \alpha^*\beta)]$ and $\theta = \pm\frac{\pi}{4} + \arg(\alpha - \beta)$. Variations of fourth-order squeezing and sixth-order squeezing with parameter x at $\delta = \pm\frac{\pi}{4}$, $\xi = 0$ and r = 0.038 have been shown in Figure 1 and Figure 2 respectively.

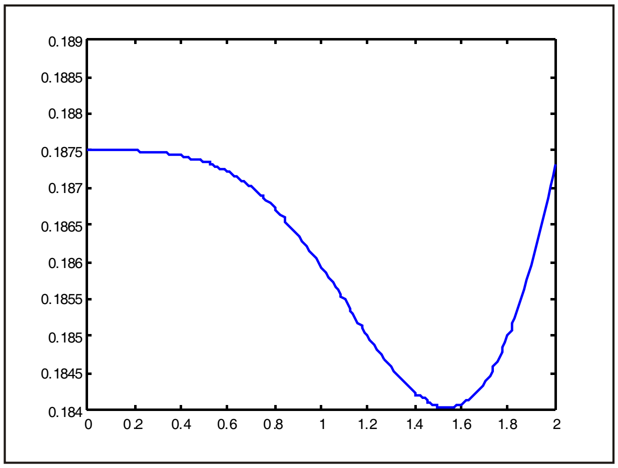


**Figure 1 : Variation of $S^{(4)} \equiv \langle \psi_2 | (\Delta X_{\delta,|\psi_2\rangle})^4 | \psi_2 \rangle$ with x at $\delta = \pm\frac{\pi}{4}$ , $\xi = 0$ and r = 0.038**

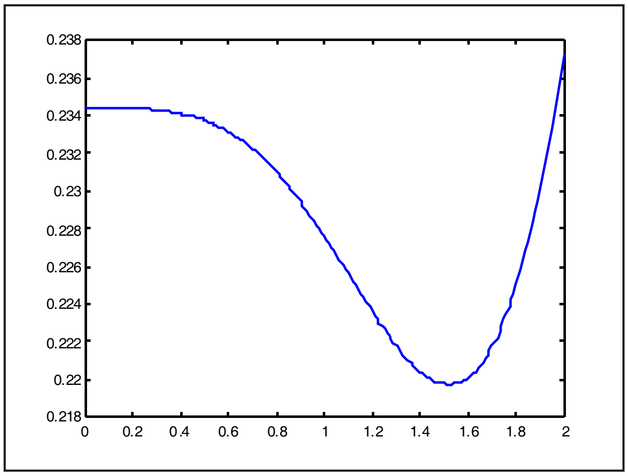


**Figure 2 : Variation of $S^{(6)} \equiv \langle \psi_2 | (\Delta X_{\delta,|\psi_2\rangle})^6 | \psi_2 \rangle$ with x at $\delta = \pm\frac{\pi}{4}$ , $\xi = 0$ and r = 0.038**

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