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HIGHER-ORDER SUB-POISSONIAN STATISTICS IN INTERACTION OF LIGHT WITH NON-ABSORBING NON-LINEAR KERR MEDIUM

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ABSTARCT

We study higher-order sub-Poissonian photon statistics of the output field of Mach-Zehnder interferometer initially in coherent state interacting with a non-absorbing non-linear Kerr medium placed into one arm of the interferometer. The interaction of optical field with Kerr medium has been modelled as an anharmonic oscillator, described by well-known interaction Hamiltonian, $H = \frac{1}{2}\lambda\, a^{+2} a^2$ **Here, the parameter** λ **is proportional to cubic non-linearity** $\chi^{(3)}$ **of the nonlinear medium, a and** a^+ **are the annihilation and creation operators for the interacting field respectively. We find that the output state of Mach-Zehnder interferometer exhibits higher-order sub-Poissonian photon statistics depending on the intensity of interacting optical field, interaction time and the reflection coefficients of the interferometer mirrors. The variations of these statistics within the optical domain of realistic values of Kerr non-linearity and intensity of interacting optical field have also been discussed.**

KEYWORDS: Non-classical Light, Higher-order Squeezing, Higher-order Sub-Poissonian Photon Statistics, Anharmonic Oscillator, Optical Kerr Effect, Mach-Zehnder Interferometer

Non-classical states (Walls, 1983; Dodonov, 2002) of optical fields, which have no classical analogues, are the subject of intense studies due to their applications in quantum information theory such as communication, quantum teleportation, dense coding and quantum cryptography etc. The non-classical effects of a quantum state are generally manifested in different ways like squeezing, higher-order squeezing, spin or atomic squeezing, polarization squeezing and various kinds of sub-Poissonian photon statistics etc. Among these non-classical effects, a well-known non-classical effect based on moments of number of photons is sub-Poissonian statistics, which is characterized by Mandel's Q parameter (Mandel, 1979) defined by

$$Q = \frac{\langle\psi|(\Delta N)^2|\psi\rangle}{\langle\psi|N|\psi\rangle} - 1\,, \qquad (1)$$

where $\Delta N = N - \langle\psi|N|\psi\rangle$ and $N = a^+ a$. The state $|\psi\rangle$ exhibits sub-Poissonian statistics for $0 < Q \le -1$, and super-Poissonian for Q > 0. The sub-Poissonian statistics has been generalized to higher-order by several authors (Kim, 1998; Erenso *et al.,* 2002). In particular, Kim defined $Q^{(n)}$, n^{th}-order Q parameter by

$$Q^{(n)} \equiv \frac{\langle\psi|(\Delta N)^{2n}|\psi\rangle - \lambda^{(2n)}(\langle\psi|N|\psi\rangle)}{\lambda^{(2n)}(\langle\psi|N|\psi\rangle)}\,, \qquad (2)$$

to generalize the sub-Poissonian photon statistics into higher orders. Here**,** $\lambda^{(2n)}(\langle\psi|N|\psi\rangle)$ represents the higher moments for the coherent state, including the vacuum. The first three expressions of $\lambda^{(2n)}(\langle\psi|N|\psi\rangle)$ are $\langle\psi|N|\psi\rangle$, $[\,3(\langle\psi|N|\psi\rangle)^2 + \langle\psi|N|\psi\rangle\,]$ and $[\,15(\langle\psi|N|\psi\rangle)^3 + 25(\langle\psi|N|\psi\rangle)^2 + \langle\psi|N|\psi\rangle\,]$ for n=1, 2, 3 respectively. According to Kim's definition, the photon statistics is called n^{th}-order sub-Poissonian if $-1 \le Q^{(n)} < 0$. The higher-order sub-Poissonian statistics in various quantum states and its applications in detection of higher-order squeezing have been studied by Prakash *et al.*, (2005).

The interaction of coherent light with a non-absorbing non-linear Kerr medium modeled as anharmonic oscillator with well-known interacting Hamiltonian has been paid to much attention because of the exactly solvable model for generation of non-classical states, which exhibits significant several non-classical effects. Here the parameter λ is proportional to cubic

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non-linearity $\chi^{(3)}$ of the nonlinear medium. Several authors have studied various non-classical effects in such interaction.

$$H = \frac{1}{2}\lambda\, a^{+^2} a^2 = \frac{1}{2}\lambda\, N(N-1)\,, \tag{3}$$

Recently Prakash *et al.* (2006) and Kumar (Kumar, 2017) have studied the problem of optimization of amplitude-squared squeezing and amplitude n^{th}-power squeezing in such interaction respectively, and reported scaling laws for these non-classical effects using an analytic estimation in the region of short interaction time and high optical power.

When a nonlinear Kerr medium is placed into one arm of the Mach-Zehnder interferometer and a phase shifter in another arm, the output states exhibit several non-classical effects (Prinova *et al.*, 1995); Chatterjee *et al.*, 2016). In the present paper, we study higher-order sub-Poissonian photon statistics of the output field of a nonlinear Mach-Zehnder interferometer initially in coherent state (Glauber, 1968) interacting with a non-absorbing non-linear Kerr medium placed into one arm of the interferometer as described in Figure 1.

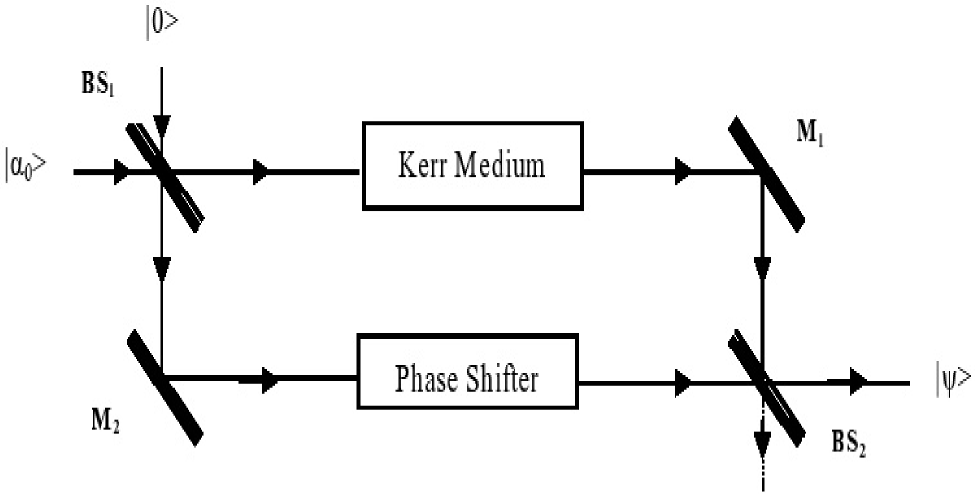

Figure 1: Schematic diagram for preparation a non-linear displaced Kerr state

HIGHER-ORDER SUB-POISSONIAN STIATISTICS IN OUTPUT STATE OF MACH-ZEHNDER INTERFEROMETER

When the field is a coherent state $|\alpha_0\rangle$ in one input to the interferometer and the vacuum state $|0\rangle$ in the other input, after the first beam splitter BS_1, the output states will be $|\psi_1\rangle = |\alpha\rangle$; $|\psi_2\rangle = |\alpha'\rangle$, where $\alpha = \sqrt{1-r_1}\,\alpha_0$, $\alpha' = \sqrt{r_1}\,\alpha_0$, r_1 is the reflectivity of the beam splitter BS_1. After passage through the Kerr medium, the state $|\psi_1\rangle$ acquires the Kerr state $|\psi_K\rangle$ at time t,

$$|\psi_K\rangle = U(t)|\alpha\rangle = e^{-i(N^2-N)\frac{\tau}{2}}|\alpha\rangle\,. \tag{4}$$

Here $U(t) = \exp(-i\,H\,t)$ is the time evolution operator, and $\lambda\, t = \tau$, the dimensionless interaction time, while the state $|\psi_2\rangle$ acquires a phase shift ϕ and becomes the coherent state $|\alpha' e^{-i\phi}\rangle$. For high reflectivity of the beam splitter BS_2, It can be shown that the output state $|\psi\rangle$ of Mach-Zehnder interferometer, known as displaced Kerr state will be

$$|\psi\rangle = D(\eta)U_K(\tau)|\alpha\rangle\,, \tag{5}$$

where $D(\eta) = \exp(\eta a^+ - \eta^* a)$ is the unitary displacement operator and $U(\tau) = e^{-i(N^2-N)\frac{\tau}{2}}$ is time evolution operator, $\eta = \sqrt{(1-r_2)}\alpha' e^{-i\phi} = \sqrt{r_1(1-r_2)}\alpha_0 e^{-i\phi}$, with r_2 being the reflectivity of BS_2.

Now using the relation $U^+(\tau)aU(\tau) = \exp\{-i\tau N\}a$ and the result, a straight forward calculations lead to

$$\langle\psi_K|a^{+n_1}a^{n_2}|\psi_K\rangle = \alpha^{(n_1+n_2)}\exp[i\frac{\tau}{2}\{n_1(n_1-1)-n_2(n_2-1)\}]\exp[\alpha^2\{-i\tau(n_2-n_1)-1\}]\,, \tag{6}$$

Also, using the properties of displacement operator, $D^+(\eta)aD(\eta)=a+\eta$, we get

$$\langle\psi|a^{+n_1}a^{n_2}|\psi\rangle = \langle\psi_K|(a^+ + \eta^*)^{n_1}(a+\eta)^{n_2}|\psi_K\rangle\,. \tag{7}$$

For simplicity of calculations, we define $P(n,n_1,n_2)$ as

$$\begin{aligned}P(n,n_1,n_2) &= \mathrm{Re}[\langle\psi_K|a^{+n_1}a^{n_2}|\psi_K\rangle\xi^{2n-n_1-n_2}]\\ &= \alpha^{n_1+n_2}|\eta|^{2n-n_1-n_2}\cos[(n_2-n_1)\phi+\alpha^2\sin\tau(n_2-n_1)+\frac{\tau}{2}\{n_1(n_1-1)-n_2(n_2-1)\}]\exp[-2\alpha^2\sin^2\frac{\tau}{2}(n_2-n_1)]\end{aligned} \tag{8}$$

Now using Eqs. (7) and (8), we have

$$\langle\psi|a^+a|\psi\rangle = P(1,1,1)+2P(1,1,0)+P(1,0,0)\,, \tag{9}$$

$$\langle\psi|a^{+2}a^2|\psi\rangle = P(2,2,2)+4P(2,2,1)+2P(2,0,0)+4P(2,1,1)+4P(2,1,0)+P(2,0,0)\,, \tag{10}$$

$$\begin{aligned}\langle\psi|a^{+3}a^3|\psi\rangle &= P(3,3,3)+6P(3,3,2)+6P(3,3,1)+2P(3,3,0)+9P(3,3,2)+18P(3,2,1)\\ &\quad+6P(3,2,0)+9P(3,3,1)+6P(3,1,0)+P(3,0,0)\end{aligned}\,, \tag{11}$$

$$\begin{aligned}\langle\psi|a^{+4}a^4|\psi\rangle &= P(4,4,4)+8P(4,4,3)+12P(4,4,2)+8P(4,4,1)+2P(4,4,0)+16P(4,3,3)\\ &\quad+48P(4,3,2)+32P(4,3,1)+8P(4,3,0)+36P(4,2,2)+48P(4,2,1)\\ &\quad+12P(4,2,0)+16P(4,1,1)+8P(4,1,0)+P(4,0,0)\end{aligned}\,. \tag{12}$$

Similarly the higher-order expectation values $\langle\psi|a^{+n_1}a^{n_2}|\psi\rangle$ for $n_1 = n_2$ can be found in the output state of Mach-Zehnder interferometer.

CONCLUSION

We study ordinary sub-Poissonian statistics and second-order sub- Poissonian statistics in the output state $|\psi\rangle$ of a non-linear Mach-Zehnder interferometer. Tedious but straight forward calculations using Eqns. (2), (9)-(12), and computer programming we get minimum values -0.8220 and -0.9630 of the parameter $Q^{(n)}$ for n=1 and n=2 at $\alpha=4$, $|\eta|=2$ and $\phi=0$ respectively. Similarly we get minimum values -0.9417 and -0.9960 of the parameter $Q^{(n)}$ for n=1 and n=2 at $\alpha=8$, $|\eta|=2$ and $\phi=0$ respectively. Variations of $Q^{(n)}$ with τ for n =1, 2 with $|\xi|=2$, $\phi=0$ at $\alpha=4$ and $\alpha=8$ have been shown in Figure 1 and Figure 2 respectively.

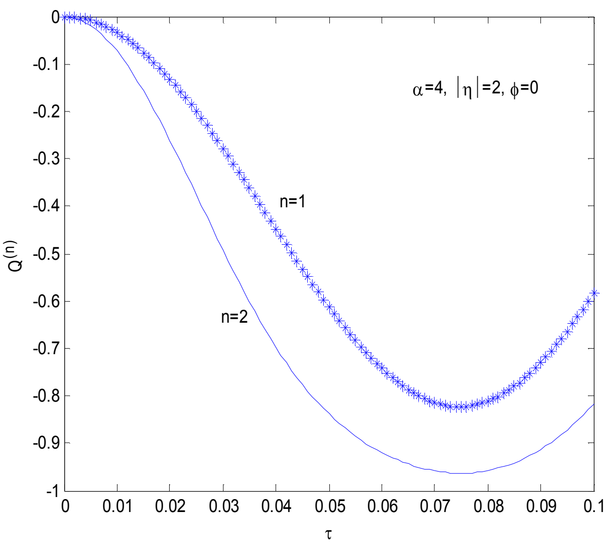

Figure 2: Variation of $Q^{(n)}$ with τ for n =1, 2 at $\alpha=4$, $|\xi|=2$ and $\phi=0$

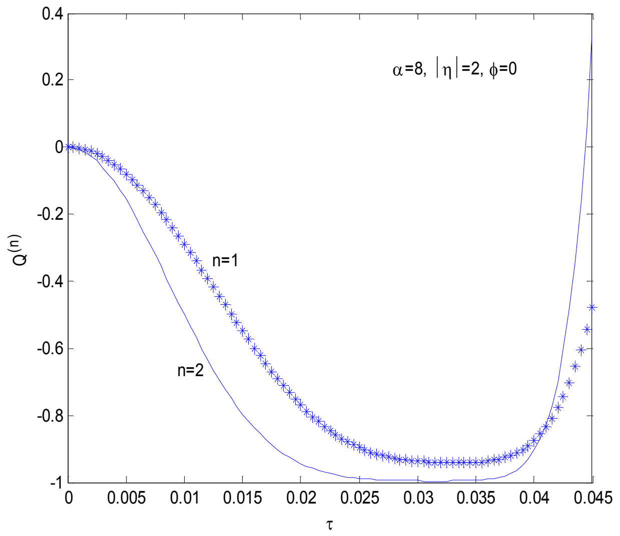

Figure 3: Variation of $Q^{(n)}$ with τ for n =1, 2 at $\alpha = 8$, $|\xi| = 2$ and $\phi = 0$

From the Figure 2 and 3, we conclude that output state $|\psi\rangle$ of a non-linear Mach-Zehnder interferometer exhibits both ordinary and second-order sub-Poissonian statistics significantly. It should also be noted from figures that n^{th}-order sub-Poissonian statistics appears more significant when we increase the order n, decrease the interaction time and increase the intensity of interacting field. In similar way, one can also show that the state $|\psi\rangle$ will also exhibit more higher-order (i.e., third, fourth, fifth order…) sub-Poissonian statistics in Mach-Zehnder interferometer.

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