Indian J.Sci.Res. 17(2): 474 - 478, 2018

ISSN: 0976-2876 (Print)
ISSN: 2250-0138(Online)

MIMO-OFDM SAR USING CIRCULARLY SHIFTED SEQUENCES

[1]Arpita Chakraborty

[1]Associate Professor, Department. Of Electronics and Communication Engineering, Aurora's Scientific, Technological and Research Academy, Hyderabad

***Abstract*:** MIMO-OFDM SAR with CP (Cyclic Prefix) has been recently proposed with distributed antenna structure, where no range-cell – interference (RCI) is present. However, in order to consecutively obtain range profiles over a large swath, multiple OFDM pulses need to be transmitted consecutively. In this paper, a CP – based MIMO-OFDM synthetic aperture radar system is proposed, where each and every transmitter transmits only a single OFDM pulse, which is used to obtain range profiles. This is done in order to obtain the same frequency band and to assure that the range resolution is not reduced. This signal is completely RCI free and can utilize full spatial diversity if the transmitting antennas are properly distributed. The main idea behind the paper is to use circularly shifted sequences as weighing coefficients in the OFDM waveform for different transmitting antennas and to apply spatial filters to convert the entire swath into small multiple sub swathes. The advantage of this technique is that each sub swath can be easily reconstructed during range reconstruction using the RCI – free range reconstruction method.

keywords : Circularly shifted sequences ,cyclic prefix , range-cell interference(RCI) , MIMO- OFDM (multiple- input multiple- output orthogonal frequency division multiplexing) , SAR(Synthetic Aperture Radar)

I. Introduction

MIMO radars have attracted much attention in recent times [1-3]. It can be observed that the MIMO – systems can operate with spatial diversity but the signals have to be orthogonal in spite of their delays temporarily, which is difficult to design and implement particularly in synthetic aperture radar (SAR) applications in range and azimuth [4-6]. Recent changes in the design of waveform in MIMO-SAR lead to generation of multiple waveforms with non - overlapping frequency bands in order to utilize full spatial diversity. But this reduces the range resolution of the radar and consequently, the range ambiguity increases. On the other hand, if the range resolution is not reduced, all the different multiple waveforms need to have the same frequency band. Otherwise their temporal versions may not be orthogonal to each other and accordingly, full spatial diversity cannot be achieved in distributed MIMO-SAR.

Orthogonal frequency division multiplexing waveforms has recently being studied for SAR applications [6, 7]. In order to obtain a group of orthogonal OFDM waveforms suitable for MIMO-SAR imaging, an interleaving frame structure in the frequency domain has been proposed [6], which divides the entire overlapping bandwidth into multiple non- overlapping sub-bands. But the above mentioned SAR imaging schemes uses matched filters, which will result in generation of side lobes. Using a cyclic- prefix at the transmitting and the receiving end , the OFDM-SAR image proposed in [8] for a single transmitter achieves ideally zero side-lobes in a particular range and can be termed as range – cell interference (RCI) free . The key idea behind for the MIMO-OFDM design is that the designed multiple waveforms should be orthogonal in the time domain, in spite of their multipath delays. However, in order to achieve the target of RCI – free reconstruction at the receiver, multiple consecutive OFDM pulses need to be transmitted in a single swath. This activity may take a longer time for a reasonable swath width.

In this paper, we propose a CP based MIMO-OFDM SAR system, where a transmitter transmits a single OFDM pulse to obtain range profiles for a swath which corresponds to the same frequency band. This ensures that the range resolution is not reduced and no RCI is generated from any transmitter and full spatial diversity can be achieved, if the transmit antennas are distributed. The main idea is to use circularly shifted sequences as the weighing coefficients in the OFDM pulses for different transmit antennas and spatial filters are used with multiple receiving antennas to divide the entire swathes into multiple sub swathes. Next each sub swath is reconstructed using the proposed RCI – free range reconstruction method. A CP based MIMO- OFDM radar for collocated transmitters is proposed in [9], for RCI –free reconstruction. This paper mainly covers distributed transmitters for RCI free range reconstruction.

II. MIMO-OFDM SAR Signal Model

Let's consider A MIMO-OFDM SAR imaging system with multiple transmit and receive antennas. CP based OFDM sequences as transmit waveforms are considered here. The sequences of MIMO SAR transmit OFDM signals are shown in Fig. 1.

[1]**Corresponding Author**

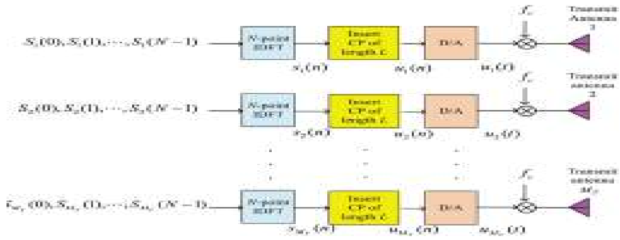

Fig. 1. MIMO-OFDM SAR Transmitters

Here it is assumed that the multiple transmit antennas are all distributive in nature. The weighing sequences for the subcarriers with length N are considered as complex in nature. The bandwidth of the generated waveform is B= NΔf , where Δf are considered as the difference in frequency between the adjacent subcarriers .

The time domain OFDM waveform can be generated by the N- point IDFT (Inverse Discrete Fourier Transform) as:

$$\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} s_m(k)\exp\left(\frac{j2\pi nk}{N}\right)\ldots\ldots\ldots\ldots\ldots (1)$$

A cyclic Prefix is added at the beginning of the transmitted signal and the length of the CP should be large enough to suppress the side lobes .In order to ensure that the transmitted energy is not unnecessarily wasted, the CP length is minimized and made equal to the maximum range cell number in a swath that will be covered by the SAR. Thus in Fig. 1, we have,

$$u_m(n) = \{s_m(n+N-L), 0 \le n \le L\}$$

$$\{s_m(n-L), L \le n \le L+N-1\}\ldots\ldots\ldots\ldots (2)$$

Let's assume that the receiving array is collocated and multiple spatial beams are formed [4], [6]. The entire swath is divided into P sub swathes. This is done by narrow receive beams which also ensures that the received echo in each of the sub swathes is significantly reduced. The receiving MIMO-SAR along with the spatial filter is shown in Fig.2.

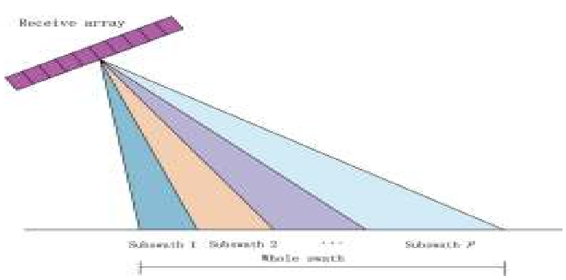

Fig. 2 MIMO-SAR With The Spatial Filters In The Receiving Mode

Assuming that the occupied range cell number of the pth sub swath is L_p and that the maximum range cell number L_o of all the sub swathes satisfies the condition,

$$L_o \le N/M_T \ldots\ldots\ldots\ldots (3)$$

where M_T is the number of transmit antennas , $L_o = \max(L_1, L_2, \ldots L_P)$. The CP length L for each transmitted OFDM waveform is $L = L_o$. After A/D conversion with sampling frequency $f_s = B$ and spatial filtering, the discrete baseband signal received from the pth sub swath can be expressed as,

$$r_p(n) = \sum_{m=1}^{M_T}\sum_{l=0}^{L_p-1} h_{p,m}(l)u_m(n-l) + v(n), 0 \le n \le N+L_p+L-1, 1 \le p \le P \quad (4)$$

where $h_{p,m}(l) = g_p(l)h_m(l)$, $h_m(l)$ and $g_p(l)$ indicates the radar cross section co-efficient of the lth range cell for the mth transmit waveform and the pth spatially filtered response respectively . The ideal response from a spatial filter can be expressed as

$$g_p(l) = \mathrm{rect}\left(\frac{l-\sum_{i=0}^{p-1} L_i}{L_p}\right)\ldots\ldots\ldots\ldots\ldots (5)$$

In practice, it is impossible to obtain an ideal response like this. If the side lobes of a spatial filter is less than -40dB, then the influence from the neighboring swathes will reduce by, at least 40dB, considering a 40-dB Taylor window in a spatial filter. In this case, the reconstructed range profile as suggested by the present method will still be RCI free for closed-space scattering points of the same sub swath and this will not be affected much due to non-ideal spatial filtering. Accordingly, the ideal spatial filter as given in Eq. (5) can be used.

III. Range Reconstruction Without Interference

Here, we remove the CP from the time- discrete received signal in the SAR, as given in eq. (4) by taking N samples starting from the lth sample point. In the range 0≤n≤N, we get:

$$z_p(n) = r_p(n+L)$$

$$= \sum_{m=1}^{M_T}\sum_{l=0}^{L_p-1} h_{p,m}(l)u_m(n+L-l) + v(n+L)$$

$$= \sum_{m=1}^{M_T}\sum_{l=0}^{L_p-1} h_{p,m}(l)s_m(n-l) + v(n+L)\ldots (6)$$

As a result of insertion of CP, $s_m(n)$ becomes periodic with a period of N as can be seen in equation (6) . Now if we carry out the N-point DFT of $z_p(n)$, it becomes

$$Z_p(k) = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} z_p(n)\exp\left(\frac{-j2\pi nk}{N}\right)$$

$$= \sqrt{N}\sum_{m=1}^{M_T} H_{p,m}(k)S_m(k) + V(k)\ldots\ldots (7)$$

where $S_m(k)$ and $V(k)$ are the N- point DFTs of $s_m(n)$ and $v(n+L)$, 0≤n≤N , respectively . $H_{p,m}(k)$ is given by

$$H_{p,m}(k) = \frac{1}{\sqrt{N}} \sum_{l=0}^{L_{p-1}} h_{p,m}(l) \exp\left(\frac{-j2\pi lk}{N}\right)$$

$$= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \bar{h}_{p,m}(n) \exp\left(\frac{-j2\pi nk}{N}\right) \ldots (8)$$

is the N- point DFTs of $\bar{h}_{p,m}(n)$ with

$$\bar{h}_{p,m}(n) = h_{p,m}(n) \ , 0 \leq n \leq L_p$$

$$= 0 \ , L_p \leq n \leq N \ldots\ldots (9)$$

Now in order to have a range- reconstruction of the swath, we have to determine the RCS coefficients $\bar{h}_{p,m}(n)$ that is present in $H_{p,m}(k)$. If it is possible to solve $H_{p,m}(k)$ for all value of k , the RCS coefficients can be found out and range reconstruction of the entire swath can be done . From equation (7), we can observe that for each values of k, there are M_T nos. of variables for $H_{p,m}(k)$. In order to solve this, we have to generate M_T nos. of equations. This can be done by sending M_T nos. of OFDM pulses and then the system would be a linear one with M_T variables. But in doing this, it would consume a large amount of time which will be detrimental for proper range reconstruction of a swath.

In this paper, only one OFDM pulse is transmitted in order to obtain the range profiles and the RCS coefficients of a swath are solved based on equation (7) for each k. This is because, after the spatial filtering with multiple receiving antennas, the RCS coefficients $\bar{h}_{p,m}(n)$ will have the property as given in equation (9). Accordingly we have to design the weighing sequences $S_m(k)$, as given in equations (1) and (7) . This is done in order to correctly determine the RCS coefficients by solving the frequency domain equations in (7) and their corresponding time-domain equations as given in (1). In order to do so, we consider two transmit antennas i.e.M_T=2. After that, discrete frequency domain matched filter is applied to equation (7) for the weighting sequences $S_m(k)$. Hence, we get

$$Y_{p,1}(k) = \frac{1}{\sqrt{N}} S_k^*(k) Z_p(k)$$

$$= S_1^*(k) \sum_{m=1}^{2} H_{p,m}(k) S_m(k) + \frac{1}{\sqrt{N}} S_1^*(k) V(k)$$

$$= H_{p,1}(k) + H_{p,2}(k)F(k) + V_1(K) \ldots\ldots\ldots\ldots (10)$$

where * denotes complex conjugate , $F(k) = S_1^*(k)S_2(k)$ and $V_1(k) = 1/\sqrt{N}\ S_1^*(k)V(k)$. Taking the N-point IDFT of the two sides of equation (10) , we get

$$\hat{h}_{p,1}(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} Y_{p,1}(k) \exp\left(\frac{i2\pi nk}{N}\right)$$

$$= \bar{h}_{p,1}(n) + \bar{h}_{p,2}(n) \otimes f(n) + \bar{v}_1(n) \ \ldots\ldots\ldots (11)$$

where f(n) and $\bar{v}_1(n)$ are the IDFTs of F(k) and V(k) respectively and ⊗ is the circular convolution . From equation (11), we can observe that if f (n) can be designed in such a way that the circular convolution of $\bar{h}_{p,2}(n) \otimes f(n)$ becomes a circular shift of $\bar{h}_{p,2}(n)$, with amount of shift being N/2 and under these circumstances the two sets of non- zero RCS coefficients $\bar{h}_{p,1}$ and $\bar{h}_{p,2}$ will not overlap. This implies that RCI-free range profiles from any channel can be obtained. As per the property of DFT, we get $F(k) = S_1^*(k)S_2(k) = \exp(j\pi k)$ i.e.

$$S_2(k) = S_1(k) \exp(j\pi k) \ldots\ldots (12)$$

With this, we get from equation (11) ,

$$\hat{h}_{p,1}(n) = \bar{h}_{p,1}(n) + \bar{h}_{p,2}(\langle n + N/2 \rangle_N) + \bar{v}_1(n) \ldots\ldots (13)$$

Similarly, the other response can be given as,

$$\hat{h}_{p,2}(n) = \bar{h}_{p,1}(n) + \bar{h}_{p,2}(\langle n + N/2 \rangle_N) + \bar{v}_1(n) \ldots (14)$$

From equations (9), (13) and (14), we get,

$$h_{p,m}(n) = \bar{h}_{p,m}(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} Y_{p,m}(k) \exp\left(\frac{j2\pi nk}{N}\right),$$
$$0 \leq n \leq N \ldots (15)$$

From the above expression we can see that the original sub swath RCS co-efficient can be considered to be RCI free. Then the whole swath RCS coefficients $h_m(n)$ can be obtained as follows :

$$h_m(n) = [h_{1,m}(n), h_{2,m}(n), \ldots\ldots\ldots\ldots\ldots h_{P,m}(n)],$$
$$m = 1, 2 \ldots (16)$$

From the range reconstruction, we can see that all the range cell scattering coefficients are recovered without any interference from other range cells. The proposed RCI free range reconstruction algorithm diagram for each sub swath is shown in Fig. 3:

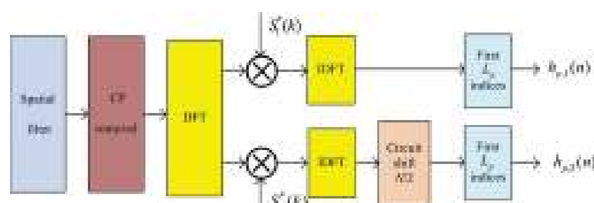

Fig. 3. MIMO radar Interference -Free Range Reconstruction

This proposed scheme can be easily extended to more transmit antennas, if equation (3) is satisfied. This can be guaranteed by dividing the entire swath into more no. of sub- swathes. This will increase the nos. of receiving antennas. In this case, the whole swath imaging can be done by putting together all the sub swath RCS coefficients $h_{p,m}(n)$ for all n , $0 \leq n \leq \sum_{p-1}^{P} L_p$ and for all m , $1 \leq m \leq M_T$, which is entirely free of RCI and it has full spatial diversity at the time of transmission .

The two weighing sequences $S_m(k)$ in equations (1) and (7) need to satisfy the property , as given in equation (12) . This is necessary to shift the RCS coefficients from the two different transmitting antennas into one sub swath but it should not overlap with each other. In addition, the orthogonality of the waveforms in the discrete frequency domain and the low peak-to- average power ratio (PAPR) in the discrete time- domain needs to be considered while it is attempt the above.

IV. Circularly Shifted Sequences

Here, it was decided to use circularly shifted sequences as the weighting sequences $S_m(k)$ that will satisfy equation (12) and have a constant modulus . Let $S_1(k)$ be a sequence , which is assumed to be circular , i.e.

$$S_1(k) = \exp\left(\frac{-j\pi\mu k(k+N_2)}{N}\right) \ldots\ldots\ldots (17)$$

where μ is an integer less than N. The discrete time – domain signal $S_1(k)$ is derived by taking its N-point IDFT . This is done as,

$$s_1(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_1(k) \exp\left(\frac{j2\pi nk}{N}\right)$$

$$= S_1^*(\mu^{-1}n) \exp\left(\frac{-j2\pi n(N_2)}{N}\right) \ldots\ldots\ldots\ldots\ldots (18)$$

It can be concluded from above, that the IDFT of the circularly shifted sequence is a constant modulus sequence. It can be inferred here that the PAPR in the time domain will be optimal in nature. From the zero periodic correlation properties of the circularly shifted sequences, we can conclude that $S_1(k)$ and $S_2(k)$ are orthogonal in nature . The advantage of the discrete- frequency domain orthogonality is that it remains unaffected by any time dispersiveness of the channel , but it is not the same with discrete – time domain othogonality which gets largely affected due to the time dispersive nature of the channel .

According to the property of the IDFT,

$$s_2(n) = \beta^* s_1(n) \exp(j\pi n) \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

which is also a constant modulus sequence . Hence, we can easily conclude that the orthogonality in the discrete frequency domain, constant modulus property in both the discrete frequency and time- domains are all satisfied by the MIMO-OFDM SAR.

V. Simulation Results

The simulation results are presented here to vindicate the proposed performance of our method. In the beginning, the interference free reconstruction with the designed circularly shifted sequence based performance of the MIMO radar is shown. The simulations are carried under the assumptions of the usage of the optimum spatial filtering. The optimal spatial filtering ensures that the antenna pattern is ideally covering the angular range of interest, thereby suppressing the side lobes, which are generated. Numbers of transmit antennas are two and the number of subcarriers are 512. The carrier frequency and the signal bandwidth of the MIMO radar are 4GHz. and 150MHz. respectively. The maximum range cell number L_o of all sub swaths is 175, which is less than N/2. Accordingly, the length of the CP is also chosen as 175.

Fig. 4(a) and 4(b) shows the reconstructed RCS scattering co-efficient from two transmit antennas without noise and with noise, respectively. This is done using the circularly shifted sequences and the interference free method, suggested earlier. There are 8 strong scattering points for both the channels, and the scattering coefficients are randomly generated.

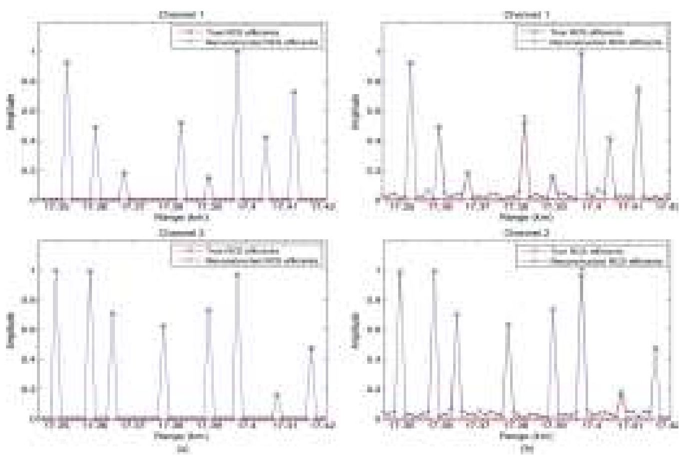

Fig. 4. Two channel reconstructed RCS scattering coefficients. Fig (a) shows without noise and Fig (b) shows with noise

Fig . 4(a) shows that both the range profiles can be easily reconstructed with perfection, when there is no noise Fig. 4(b) shows that the proposed method also has a good performance in a noisy environment, without interference from any channel. In Fig.5, o denotes the amplitude of the true range profile and +,* and x denote the reconstructed range profiles of the proposed method, MIMO and OFDM chirp waveforms respectively. Since in the proposed method, there is no range cell interference between the scatterers in different range cells, the range profile is perfectly recovered as can be seen in Fig. 5(a). Fig 5(b) and 5(c) shows the reconstructed range profiles of MIMO chirp and OFDM chirp waveform respectively. It can be inferred from these two waveforms that using matched filtering results in inaccuracies in the peak amplitudes, which may damage the range profile. In this way the proposed range cell interference free method is superior to the methods as shown in Fig 5(b) and Fig. 5(c).

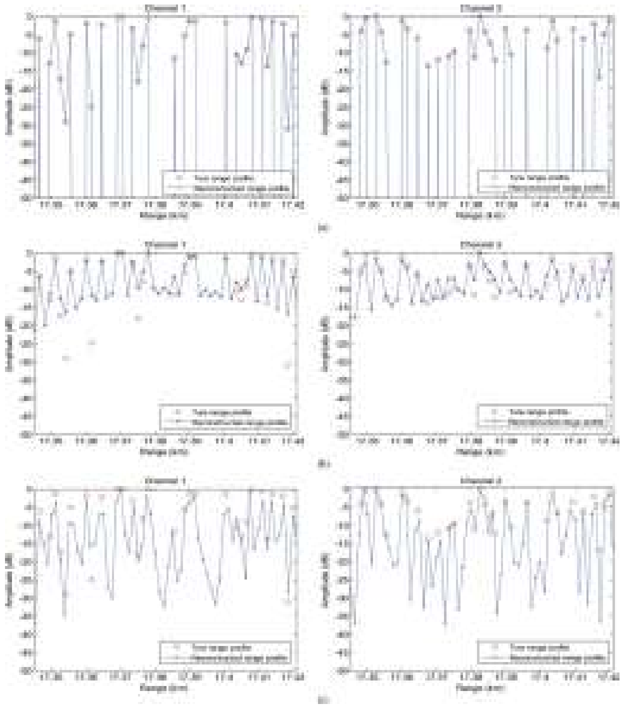

Fig 5 . Reconstructed range profiles. (a) Proposed range cell interference free method, (b) MIMO chirp waveform and (c) OFDM chirp waveform both with matched filters.

The difference between the true range profile and reconstructed range profile is almost negligible. During the course of range reconstruction, the each swath coverage is done by adding the nos. of sub swathes. During the addition process, some amount of interferences from the scatterers in the sub swathes may result in the difference. This is clearly evident in Fig. 5 from the fact that the proposed method had a considerable amount of improvement, as compared to the MIMO chirp and OFDM chirp waveform. These differences can also arise due to improper positioning of the antenna.

VI. Conclusion

In this paper, a CP based MIMO-OFDM SAR system have been proposed. Here, each and every transmitter transmits a single OFDM pulse to obtain range information for a swath and has the same bandwidth. A circularly shifted sequence has been proposed as the weighting coefficients for the OFDM waveforms from different transmitters and spatial filters are used with multiple receive antennas. This is done in order to divide the entire swath into multiple sub swathes and then each sub swath is reconstructed using the proposed interference free range – reconstruction method. Using this technique, spatial diversity can be achieved by obtaining multiple channel range profiles from different transmitters. Superior performances can be achieved with this RCI- free range reconstruction method as compared to MIMO chirp and OFDM chirp techniques, as is evident in the waveforms.

References

[1] E. Fishlert et al.(2004). MIMO radar: An idea whose time has come. *Proc.IEEE Radar Conf*: 71–78.

[2] D.W. Bliss and K.W. Forsythe.(Nov. 2003). Multiple-input multiple-output (MIMO)radar and imaging: Degrees of freedom and resolution. *Conf. Rec. 37thAsilomar Conf. Signals. Syst. Comput*: Pacific Grove, CA, US. vol.1.54–59.

[3] J. Li and P. Stoica.(Oct 2008). MIMO Radar Signal Processing .*Wiley* Hoboken, NJ, USA

[4] G. Krieger.(May 2014). MIMO-SAR: Opportunities and pitfalls. *IEEE Trans.Geosci . Remote Sens*. vol. 52. no. 5 . 2628–2645

[5] W. Wang.(Sep. 2014). *MIMO SAR chirp modulation diversity waveform design* .IEEE Geosci. Remote Sens. Lett. vol. 1. no. 9. 1644–1648,

[6] J. Kim, M. Younis, A. Moreira, and W. Wiesbeck.(May 2013). A novel OFDM chirpwaveform scheme for use of multiple transmitters in SAR. *IEEE Geosci.Remote Sens. Lett*., vol. 10, no. 3 . 568–572

[7] D. Garmatyuk and M. Brenneman.(Oct.2011). Adaptive multicarrier OFDM SARsignal processing. *IEEE Trans. Geosci. Remote Sens*. vol. 49, no. 10.pp. 3780–3790.

[8] T. Zhang and X.-G. Xia.(Jan 2015). OFDM synthetic aperture radar imaging with sufficient cyclic prefix. *IEEE Trans. Geosci. Remote Sens*. vol. 53, no. 1.pp. 394–404.

[9] Y. Cao, X.-G. Xia, and S. Wang.(2014).IRCI free co-located MIMO radar based on sufficient cyclic prefix OFDM waveforms. Unpublished. [Online]. Available: http://arxiv.org/pdf/1406.1488 .