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LOCALLY ROTATIONALLY SYMMETRIC BIANCHI TYPE-I MODEL WITH VARIABLE DECELERATION PARAMETER

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ABSTRACT

Locally rotationally symmetric (LRS) Bianchi type I cosmological model of the universe have been studied in the cosmological theory. A new class of exact solutions has been obtained by considering variable deceleration parameter.

KEY WORDS : LRS Bianchi type model, variable deceleration parameter, cosmology

Einstein's field equations are a coupled system of highly nonlinear differential equations and we seek physical solutions to the field equations for applications in cosmology and astrophysics. In order to solve the field equations, we normally assume a form of the matter content or suppose that space time admits killing vector symmetry Kramer and Schmutzer (1980). Solution to the field equations may also be generated by applying a law of variation for Hubble's parameter. In simplest case the Hubble law yields a constant value for the deceleration parameter. It is worth observing that most of the well know models of Einstein's theory and Brans-Diske theory with curvature parameter k = 0 including inflationary models, are models with constant deceleration parameter. In earlier literature cosmological models with a constant deceleration parameter have been studied by several authors Kramer and Schmutzer (1980), Berman and Gomid (1983), Maharaj and Naidoo (1993). But red-shift magnitude test has had a chequered history. During the 1960s and the 1970s,it was used to draw very categorical conclusions. The deceleration parameter q_0 was then claimed to lie between 0 and 1 and thus it was claimed that the universe is decelerating. Todays situation , we feel, is hardly different. Observations Riess et al.(2004) of type Ia supernovae(SNe) allow to probe the expansion history of the universe. The mai conclusion of these observations is that the expansion of the universe is accelerating. S0 we can consider the cosmological models with variable deceleration parameter. The readers are advised to see the papers by Vishwakarma and Narlikar (2005) and references the rein for a review on the determination of the deceleration parameter from super novae data.

Pradhan and Otarod (2006) have studied the universe with time dependent deceleration parameter in presence of perfect fluid. Motivated by the recent results on the BOOMERANG experiment on cosmic Microwave Background Radiation by Bernardis (2000).

In this latter we investigate an LRS Bianchi type-I model by taking the deceleration parameter to be variavle. First we present the basic equations of the models and the solutions of fields equations of Sen (1957) , Sen and Dunn (1971). Then we discuss the models and present our results.

Metric and Field equation : We consider the LRS Bianchi type 1 metric

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 (dy^2 + dz^2) \tag{1}$$

where A and B are functions of time t.

The energy momentum tensor for a perfect fluid is

$$T_{ij} = (\rho + p) v_i v_i + p g_{ij} \tag{2}$$

With equation of state

$$p = \omega\rho , \qquad 0 \le \omega \le 1 \tag{3}$$

where p and ρ are pressure and energy density respectively and υ_i is the unit flow vector satisfying $v_i v^i$= -1, Einstein's field equation with constant G(constant gravitational) is given by

$$R_{ij} - \tfrac{1}{2} R g_{ij} = -8\pi G T_{ij} \tag{4}$$

We now assume that the law of conservation of energy $T^i_{j;j} = o$ gives

$$\rho + (\rho + p) \quad (\frac{A}{A} + \frac{2B}{B}) = 0$$

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or $$\rho+(\rho+p)(\frac{3\dot{R}}{R}) = 0 \tag{5}$$

Using equation (3) in (5) we obtain

$$\rho = \frac{K^3}{R^3(\omega+1)} \tag{6}$$

Einstein's field equation (4) for the metric (1) leads to

$$2\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2} = -8\pi Gp \tag{7}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB} = -8\pi Gp \tag{8}$$

$$2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2} = 8\pi G\rho \tag{9}$$

Where dots on A, B denote the ordinary differentiation with respect to t and G is the gravitational constant.

Eliminating p and G from equation (7) and (8), we obtain

$$\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)^{\cdot}+(\frac{\dot{A}}{A}-\frac{\dot{B}}{B})(\frac{\dot{A}}{A}+\frac{2\dot{B}}{B}) = 0 \tag{10}$$

which on integration gives

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B} = \frac{K}{R^3} \tag{11}$$

where K is a constant and $R^3 = AB^2$

$H = \frac{\dot{R}}{R} = \frac{\theta}{3}$, θ is the volume expansion scalar

Equation (6), (7), (8) and (9) can also be written in terms of Hubble parameter H, deceleration parameter q, shear and scale factor R as

$$\rho = k_3 R^{-3(1+\omega)} \tag{12}$$

$$H^2(2q-1)-\sigma^2 = 8\pi G p \tag{13}$$

$$3H^2-\sigma^2 = 8\pi G\rho \tag{14}$$

Where $q = \frac{-R\ddot{R}}{\dot{R}^2}$ and $\sigma^2 = \frac{K_1^2}{3R^6}$

from equation (14), we obtain

$$\frac{3\sigma^2}{\theta^2} = 1-24\pi G\rho$$

which implies that for $8\pi G=1$, $G>0$

$$0<\frac{\sigma^2}{\theta^2}<1/3, 0<\frac{\rho}{\theta^2}<\frac{1}{3}$$

therefore the presence of positive G lowers the upper limit of anisotropy.

From equation (13) and (14) we obtain

$$\dot{\theta} = -3\sigma^2-\frac{24\pi G(\omega+1)\rho}{2}$$

$$= \frac{3}{2}(\omega-1)\sigma^2-\frac{9}{2}(\omega+1)H^2 \tag{15}$$

It means that the rate of volume expansion decreases during time evolution and presence of negative G is to halt this decrease whereas the positive G promotes it.

SOLUTION TO THE FIELD EQUATION

We consider the deceleration parameter to be variable

$$q = -\frac{\ddot{R}R}{\dot{R}^2} = b \text{ (variable)}$$

above equation may be rewritten as

$$\frac{\ddot{R}}{R}+b(\frac{\dot{R}}{R})^2 = 0 \tag{16}$$

The general solution of equation (16) is given by

$$\int e^{\int \frac{b}{R} dR} dR = t+n \tag{17}$$

Where n is integrating constant.

In order to solve the problem completely, we have to choose $\int \frac{b}{R}\, dR$ in such a manner that equation (17) be integrable.

Without any loss of generality we consider

$$\int \frac{b}{R}\, dR = I_n L(R) \tag{18}$$

Which does not effect the nature of generality of solution. Hence from equation (17) and (18) we obtain

$$\int L(R)\, dR = t+n \tag{19}$$

Let us consider $L(R) = \frac{1}{2k_1\sqrt{R+k_2}}$, where k_1 and k_2 are constants.

Inserting the value of L (R) into (19) then integrating we obtain

$$R(t) = \alpha_1 t^2+\alpha_2 t+\alpha_3$$

We take = $\alpha_1 = \alpha_2 = 1$ and $\alpha_3 = 0$

$$R(t) = (t^2+t) \tag{20}$$

For this solution metric (1) assumes the form

$$ds^2 = - dt^2 + (t^2 + t)^2 [k_2^2 \, e^{\frac{4k}{3} f(t)} dx^2 + k_1^2 e^{-2kf(t)} (dy^2 + dz^2)] \quad (21)$$

where $f(t) = \int \frac{dt}{(t^2 + t)^3}$

for the model (21), Hubble parameter H, expansion scalar θ, shear scalar σ, deceleration parameter q , the spatial volume V, cosmological energy density ρ, and pressure p are given by

$$H = \frac{R}{R} = \frac{2}{(t+1)} + \frac{1}{t(t+1)} \quad (22)$$

$$\theta = 3H = \frac{6}{(t+1)} + \frac{3}{t(t+1)} \quad (23)$$

$$\sigma^2 = \frac{k^2}{3(t^2+t)^6} \quad (24)$$

$$q = \frac{-2(t^2+t)}{(4t^2+4t+1)} \quad (25)$$

$$V = k_2 k_1^2 (t^2+t)^3 e^{-\frac{4k}{3} f(t)} \quad (26)$$

$$p = \omega \rho = \frac{\omega k_3}{(t^2+t)^{3(\omega+1)}} \quad (27)$$

Using equation (22), (24) and (27) in (14) we obtain

$$8\pi G k_3 = \frac{3(2t+1)^2}{(t^2+t)^{-(1+3\omega)}} - \frac{k^2}{3(t^2+t)^2} \quad (28)$$

DISCUSSION

In the model, we observe that the spatial volume V →0 as t → 0 and expansion scalar θ → ∞ as t → 0 which shows that the universe starts evolving with zero volume and infinite rate of expansion. The scale factor also vanish at t = 0 and hence the model has a point type singularity at the initial epoch. The cosmological energy density ρ, pressure p and shear scalar σ are approach to infinite as t → 0.

With t increases the expansion scalar and shear scalar decrease but spatial volume increases. As t increases all the parameters ρ, p, and θ decrease and tend to zero asymptotically. Therefore, the model essentially gives an empty universe for large value of t .The gravitational term G is decreasing function of cosmic time t provided, $(1+3\omega) \notin [0, 2]$. The ratio $\frac{\sigma}{\theta} \to 0$ as $t \to \infty$, which shows that the model approaches isotropy for the large value of t.

CONCLUSION

In this paper we have studied a spatially homogeneous and isotropic LRS Bianchi type-I space time with the variable deceleration parameter. Einstein's field equations have been solved. Expressions for some important cosmological parameters have been obtained and physical behaviour of the models are discussed in detail, clearly the model represent shearing, non-rotating and expanding models with a big-bang start. The models have point type singularity at the initial epoch and approach isotropy at late times. Finally the solutions presented here are new and useful for a better understanding of the evolution of the universe in the LRS Bianchi type-I universe with variable deceleration parameter.

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