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GENERATION OF STANDARD NORMAL RANDOM VARIABLES

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ABSTRACT

In this paper, we discuss generation of random variables from standard normal distribution. We apply inverse transform method to approximation of cumulative normal distribution. We require atleast two uniform random variables to generate a single standard normal variable, but in the proposed algorithm only a single uniform variable is enough to generate the standard normal variables. We test whether the generated sample possess the characteristics of standard normal distribution and the randomness of the observations.

KEY WORDS: Normal distribution, bootstrap confidence intervals, central limit theorem, autocorrelation function

Simulations requiring Gaussian random numbers are critical in fields including communications, financial modeling, and many others. Probably the most often used nonuniform distribution are those of the normal family with mean μ and variance σ^2 which denote as N($\mu\sigma^2$). A random variable from the standard normal distribution, N(0,1) can easily be transformed so as to have N($\mu\sigma^2$) distribution. Hence, we only consider generation of variates from the standard normal distribution. The following are the methods to generate the standard normal variates (Thomas et al., 2007 and Wetheril, 1965).

Method-1: Central Limit Theorem Approximation

Use of the central limit theorem on U(0,1) random variables provide a simple method for closely approximating normal random variates. If $U_1, U_2, ... U_n$ are independently distributed as U(0,1). Then

$$Z = \frac{\left(\sum U_i - \frac{n}{2}\right)}{\sqrt{n/12}}$$ has an

approximate N(0,1) distribution. Choosing n=12 leads to the simple form. $Z=\sum U_i-6$ This method requires many uniform random variables to generate a single random variable from the standard normal distribution.

Method-2: Box-Muller Method

Another method that is also very easy to implement was introduced by Box-Muller (1958). This algorithm can be used for generating a pair of independent random variables from the standard normal distribution.

1. Generate two independent random numbers U_1 and U_2 from U(0,1) distribution.
2. Return $Z_1 = \sqrt{-2\ln U_1}\, Cos(2\pi U_2)$ and

$$Z_2 = \sqrt{-2\ln U_1}\, Sin(2\pi U_2)$$

This algorithm requires two uniform variables to generate a single standard normal variable.

Method-3: Polar Method

A disadvantage of the Box-Muller algorithm is the necessity to compute sine and cosine functions, which is time consuming for a computer. The following algorithm uses essentially the same transformation as the Box-Muller algorithm. However, it avoids the computation of the sine and cosine functions.

1. Generate two random numbers U_1 and U_2 from U(0,1) distribution.
2. Set $V_1 = 2U_1 - 1, V_2 = 2U_2 - 1$ and $S = V_1^2 + V_2^2$.

Note that V_1 and V_2 are U(-1,1).

3. If S>1, go to step 1, otherwise go to step 4.
4. Return the independent standard normal variables

$$Z_1 = \sqrt{\frac{-2\ln S}{S}}V_1 \text{ and } Z_2 = \sqrt{\frac{-2\ln S}{S}}V_2$$

This algorithm also requires two uniform variables to generate a single standard normal variable.

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Proposed Method

Unfortunately, there is no closed form solution available for the cumulative distribution function (CDF) of standard normal distribution. A number of approximate functions for a cumulative normal distribution have been reported in the literature. Many approximations of CDF of standard normal distribution contain either more number of constants in the model or a time consuming complicated functions (Thomas et al., 2007). A simpler logistic approximation of the cumulative normal distribution is given by Bowling et al. (2009). The best logistic fit for the CDF of standard normal distribution is given by.

$$\phi(Z) = \frac{1}{1+e^{-1.702Z}}$$

This approximate function can be used to generate the random numbers from the standard normal distribution using the inverse transform method (Jhonson, 1987). The following algorithm is used to generate the standard normal variates.

1. Generate U from the U(0,1) distribution.

2. Return $Z = \dfrac{-\ln\left(\dfrac{1}{u}-1\right)}{1.702}$.

This algorithm requires only a single uniform random variable to generate the standard normal variables and it is very much easier to apply and compute.

Empirical Study

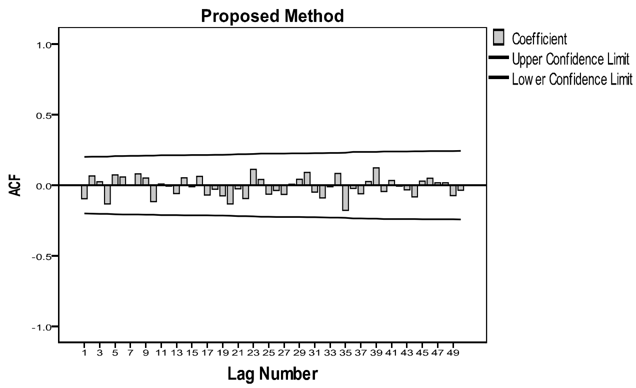
We generate the random numbers using the above procedures. We test the characteristics of the normal distribution such as $\mu=0$, $\sigma_2=1$, $\beta_1=0$ and $\beta_2=3$ To test the above characteristics, we consider the bootstrap confidence intervals (Becher et al., 1993; Efron and Tibshirani, 1993) for the parameters of mean, variance, skewness and kurtosis. If the confidence intervals consists the above parameter values then the generated sample can be treated as a normal sample. We use the autocorrelation function to test the randomness of the values. Generation of the random samples is carried out on Microsoft Excel software and the calculation of descriptive statistics along with the bootstrap confidence intervals and the autocorrelation functions are computed using the PASW-18 software. The following table gives the descriptive statistics of the sample of size 100 generated from the proposed method.

Table 1: Descriptive Statistics

	Statistic	Std. Error	Bootstrap (10000 iterations)			
			Bias	Std. Error	95% Confidence Interval	
					Lower	Upper
N	100		0	0	100	100
Range	8.35					
Minimum	-3.46					
Maximum	4.89					
Mean	-0.007		0.001	0.107	-0.216	0.209
Variance	1.170		-0.010	0.276	0.707	1.775
Skewness	0.588	0.241	-0.153	0.710	-0.851	1.677
Kurtosis	4.019	0.478	-0.918	2.089	-0.556	6.659

From the above table it is observed that $\mu=0\in(-0.216, 0.209)$, $\sigma^2=1\in(0.707, 1.775)$, $\beta_1=0\in(-0.851, 1.677)$ and $\beta_2=3\in(-0.556, 6.659)$ and . Therefore the generated sample using the proposed method satisfying the characteristics of the standard normal distribution and it can be considered as a sample drawn from standard normal distribution. We compute the autocorrelation function to check whether the generated values are random or not. The following figure gives us the sample autocorrelation function.

Figure 1: Sample Autocorrelation Function



As the results indicate, none of these autocorrelations is significantly different from zero at the level 0.05. This proves that the generated sample is random.

CONCLUSION

From the above study, it is observed that the proposed method requires only a single uniform random variable to generate the standard normal variables. The proposed method generates the samples at random from the standard normal distribution. Comparing to the other methods, the proposed method is easy to apply and compute.

REFERENCES

Becher H., Hall P. and Wilson S.R., 1993. Bootstrap hypothesis testing procedures. Biometrics, **49**(4): 1268-1272.

Bowling S.R., Khasawneh M.T., Kaewkuekool S. and Cho B.R., 2009. A logistic approximation to the cumulative normal distribution. Journal of Industrial Engineering and Management, **2**(1):114-127.

Box G.E.P and Muller M.E., 1958. A note on the generation random normal deviates. Annals of Mathematical Statistics, **29**: 610-611.

Efron B. and Tibshirani R.J., 1993. An introduction to the Bootstrap, Chapman and Hall, New York.

Johnson M.E., 1987. Multivariate Statistical Simulation. John Wiley and Sons, New York.

Thomas D.B., Luk W., Leong P.H.W. and Villasenor J.D., 2007. Gaussian random number generators. ACM Computing Surveys, **39(**4) Article 11.

Wetherill G.B., 1965. An approximation to the inverse normal function suitable for the generation of random normal deviates on electronic computers. Journal of the Royal Statistical Society Series C, **14**: 201-205.