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APPROXIMATE ANALYSIS OF STRENGTH OF TEHRAN'S TALL BUILDINGS REINFORCED WITH SPANDREL FRAME AGAINST LATERAL FORCES

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ABSTRACT

Spandrel frame system is one of the most applicable resistant systems against lateral forces in tall buildings of Tehran. Usual analysis of these frames is often costly and time-consuming due to involving numerous degrees of freedom. In this paper a method for investigation of shear lag and estimation of stress in constitutive elements of spandrel frame is presented. In this method, with consideration of deformation functions for flange and web frame and then writing stress-deformation function and using the principle of least energy, functions are presented for lateral and vertical displacement of structure. Proposed relations of this study are capable to consider shear lag phenomenon for both flange and web frame. Through consideration of variation of shear lag with elevations, factors are presented as functions of elevation through numerical solution of several structures.

KEYWORDS: Analysis of structure strength, spandrel frame, tall buildings

The framed-tube system is regarded as an economic solution to control lateral displacement of high-rise buildings. Framed tube is a set of beams and columns connected as a framework and enclose structure perimeter. The main philosophy of designing spandrel structures for very tall buildings is to use resistant material as long as possible against lateral loads in peripheral of structure so that lateral solidity of system reaches to its maximum. The most efficient structure is constructed when spandrel columns are connected anyway to act as cantilever closed-wall box. Spandrel frame involves exterior columns near to each other, connected by deep lateral main beams in each story and constitute a leaky closed wall section.

Spandrel frame acts as box beam under lateral loads such as wind and earthquake. Deflection and bending moment caused by lateral loads due to axial stresses are endured by columns located in four sides of the building. Shear due to these lateral loads are endured by bending in columns and beams in both sides of the building. Deformations due to shear lag may have adverse impact on secondary non-loading bearing elements of a building. Shear lag causes deflection in the plane of each story that led to intensification of existing deformation and stresses. Using available softwares for analyzing three-dimensional structures is the most accurate analysis method. This method entails a lot of time because accurate modeling and analysis of these structures is very costly and time-consuming. Therefore other approximate methods are proposed that are very useful in estimation of stresses and preliminary design of structure.

Spandrel frame structure as box beam is proposed in (Sabouni and Al-Mourad, 1997) through numerical studies on many structures and effective width of flange is the least value between two values of half width of web frame or ten percent of structure height. Various simulation methods are proposed in (Cimpoeru and Murray, 2003) (Fan et al., 2009) (Halabieh and Tso, 2003) that consider elastic behavior of spandrel frame as equivalent shells. According to Kwan method, shear lag is considered separately in flange and web frame because shear lag in a plane is predominately related to properties of that plate (Shah and Ribakov, 2011). Two phenomena of positive and negative shear lag complicate behavior of these frame that occur in below and above of a structure, respectively. The approximate methods that already have been presented don't account these phenomena in flange and web frame clearly or account them with significant errors (Kobielak and Tatko, 2010). Deformations related to shear lag can affect adversely on non-loading bearing elements of a building and led to deflection in plane of each story and intensification of existing deformation and stresses (Luo et al., 2004). These intensifications endanger safety and stability of structure. In this paper, through consideration of separate deformation function for flange and web frame and then writing stress-deformation relations and using principle of least energy, function are proposed for lateral

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and vertical displacement.

MODELING METHODS

Modeling of frame planes were performed as orthotropic equivalent shells in a manner that spandrel frame can be analyzed as a continuous structure. Spandrel frame structure shown in figure 1 can be considered as a composition of two plate of flange frame parallel to direction of lateral loads and two plate of web frame perpendicular to direction of lateral loads, regarding following assumptions:

1. Out-planar behavior in comparison with planar behaviors of frames is negligible regarding rigidity of roofs
2. Dimensions and distances of beams and columns are the same so that frame plane can be equivalent to integrated orthotropic shell (Wang and Chan, 2006).

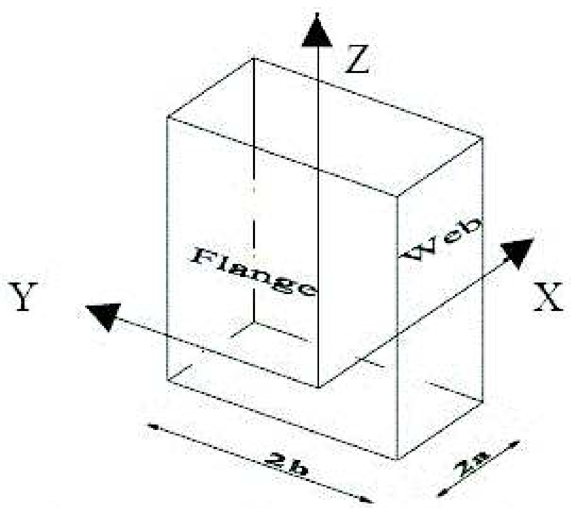

Figure 1: Orthotropic plane of spandrel frame

MATHEMATICAL RELATIONS

Shear lag occur both in flange plans and web plans, so distribution of axial stresses in flange and web plans isn't linear and simple. In this study, distribution of axial deformation in flange and web is considered as quadratic and cubic functions respectively in a manner that intensity of axial stresses in web depends on intensity of axial stresses in web. Due to shear lag phenomenon, planar section after loading is no longer flat. Axial deformations in web and flange plates are shown with w and w' respectively that are given by following equations:

$$W = \phi\alpha\left[(1-\frac{\alpha}{\lambda})\frac{x}{a}+\frac{\alpha}{\lambda}(\frac{x}{a})^3\right] \quad ....(1)$$

$$W' = \phi\alpha\left[(1-\frac{\beta}{\lambda})+\frac{\beta}{\lambda}(\frac{y}{b})^2\right] \quad ....(2)$$

In above relations, φ is deflection of planar section that connects 4 corner of tubular structure which initially located on horizontal plate and α and β are dimensionless factors of shear lag that represent degrees of shear lag in flange and web plates. λ is variation factor of shear lag with elevation. Relation of deflection section φ, axial strains and shear strains in flange and web plates are given by following expressions, respectively:

$$\phi = \frac{1}{EI}\int_0^z M dz \quad ....(3)$$

$$\varepsilon_z = \frac{\partial W}{\partial z} \quad ....(4)$$

$$\varepsilon_z' = \frac{\partial W'}{\partial z} \quad ....(5)$$

$$\gamma_{xz} = \frac{\partial U}{\partial z}+\frac{\partial W}{\partial x} \quad ....(6)$$

$$\gamma_{yz} = \frac{\partial W'}{\partial y} \quad ....(7)$$

According to above relations, strain energy of spandrel frame is given by following relation:

$$\Pi_e = \int_0^h \int_0^a t_w (E\varepsilon_z^2 + G\gamma_{xz}^2) dx dz + \int_0^h \int_0^b t_f (E\varepsilon_z^2 + G\gamma_{yz}^2) dy dz \qquad ....(8)$$

On the other hand, potential energy of applied lateral load is given by following relations in which, *U*(*z*) is lateral displacement of structure.

First mode: point load P above structure:

$$\Pi_p = -PU(h) \qquad ....(9)$$

Second mode: load with monotonous distribution and intensity P in unit of elevation:

$$\Pi_p = -\int_0^h PU(z)dz \qquad ....(10)$$

Third mode: load with linear distribution (rectangular) with intensity T in unit of elevation in above and intensity of zero in below:

$$\Pi_p = \int_0^h T \frac{Z}{h} U(z)dz \qquad ....(11)$$

Using principle of least energy, α and β for relations (1) and (2) are determined and λ relations have been obtained from numerical solution of several structures

$$\alpha = \alpha_1 \left(1 - \frac{Z}{H}\right)^2 + \alpha_2 \left[2\frac{Z}{H} - \left(\frac{Z}{H}\right)^2\right] \qquad ....(12)$$

$$\beta = \beta_1 \left(1 - \frac{Z}{H}\right)^2 + \beta_2 \left[2\frac{Z}{H} - \left(\frac{Z}{H}\right)^2\right] \qquad ....(131)$$

Values of α1, α2, β1 and β2 are listed in table (1):

Table 1:Values α and β of shear lag factors (Rutenberg and Eisenberger, 1983)

Load type	α	β
Single load above structure	$\alpha_1 = \frac{1.17m_w + 1.00}{m_w^2 + 2.67m_w + 0.57}$ $\alpha_2 = \frac{0.29m_w + 1.00}{m_w^2 + 2.67m_w + 0.57}$	$\beta_1 = \frac{3.5m_f + 12.60}{m_f^2 + 11.20m_f + 10.08}$ $\beta_2 = \frac{0.88m_f + 12.60}{m_f^2 + 11.20m_f + 10.08}$
Monotonous load	$\alpha_1 = \frac{2.57m_w + 1.12}{m_w^2 + 2.94m_w + 0.64}$ $\alpha_2 = \frac{0.03m_w + 1.12}{m_w^2 + 2.94m_w + 0.64}$	$\beta_1 = \frac{7.72m_f + 14.15}{m_f^2 + 12.35m_f + 11.32}$ $\beta_2 = \frac{0.08m_f + 14.15}{m_f^2 + 12.35m_f + 11.32}$
Load with linear distribution	$\alpha_1 = \frac{2.22m_w + 1.09}{m_w^2 + 2.86m_w + 0.62}$ $\alpha_2 = \frac{0.10m_w + 1.09}{m_w^2 + 2.86m_w + 0.62}$	$\beta_1 = \frac{6.67m_f + 13.71}{m_f^2 + 12.01m_f + 10.97}$ $\beta_2 = \frac{0.29m_f + 13.71}{m_f^2 + 12.01m_f + 10.97}$

$$m_w = \frac{G_w H^2}{E_w a^2} \qquad ....(14)$$

$$m_f = \frac{G_f H^2}{E_f b^2} \qquad ....(15)$$

It can be concluded from these relations that 1) shear lag factors of each panels is depends on elastic properties of those panels and structure height and 2) shear lag factors increase with increment of structure dimensions.

Calculation of Shear Lag Variation Factor with Height (λ)

Phenomenon of shear lag occurs not only in spandrel frames but also in box-shaped cantilever beams (Luo et al., 2004). Numerical solution was used for calculation of shear lag variation factor with height. According to above-mentioned method, values of α and β is independent from λ and are equal with the same values presented in table ,1.

Therefore, using Hook's Law, we have:

$$\sigma_{web} = E\frac{\partial W}{\partial Z} \qquad ....(16)$$

$$\sigma_{flange} = E\frac{\partial W'}{\partial Z} \qquad ....(17)$$

In equations (16) and (17), values of α and β are substituted from relations (12), (13) and table 1 and finally we have very long relations for stresses. Each of six equations has 3 unknown parameter involving z (elevation variable) and x or y as longitudinal variables of frame for flange, web and λ shear lag factor with elevation. For instance, figure 2 shows analysis result of five structures for monotonous spread load.

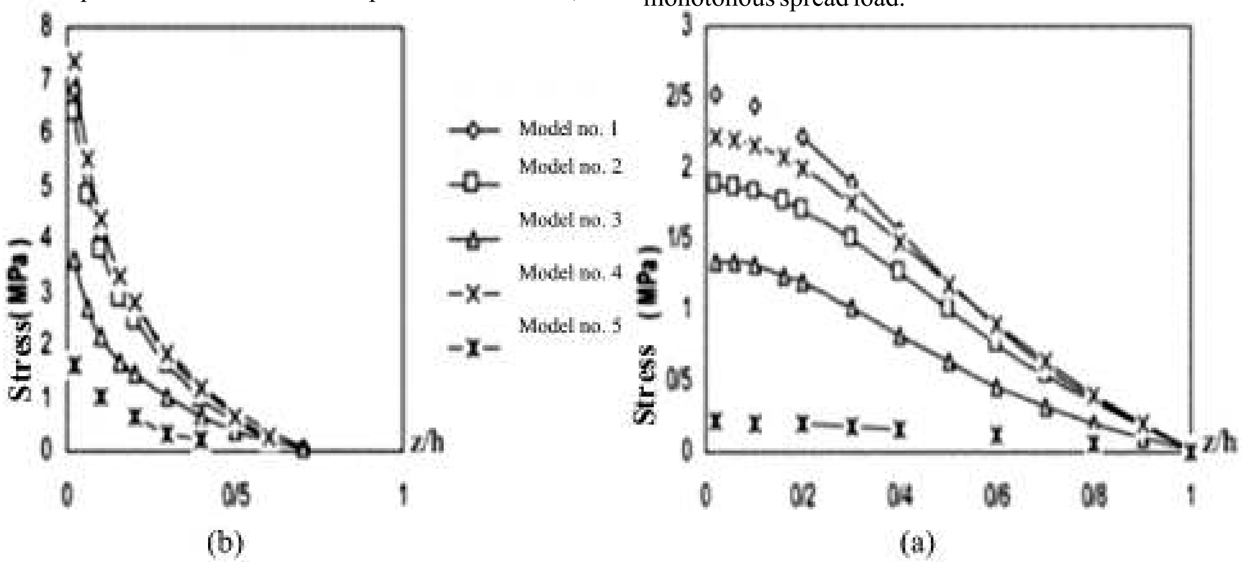

Figure 2: stress- a) in middle point of flange frame, b) crossing point of flange frame and web frame for monotonous load

For calculation of λ, it is required that several various structures must be analyzed accurately for each type of load and accurate stresses of various points of frame in various levels of each of these structures to be obtained. With accurate results of analyses, it is possible to solve each of these relations for stress in point (x, y) for flange and point (y, z) for web and find intended value of λ. Several various structures have been considered for various modes of loads in about 12 elevation levels and five points have been considered in each section. By equalizing values of stresses from relations (16) and (17) with values of actual stresses resulted from accurate analysis, values of λ have been obtained that after sorting them based on z/h nonlinear regression, functions have been presented for both flange and web frame that are listed in tables 2 and 3. According to figure 3 it can be indicated form stress distribution in flange and web frame that in drawing diagram of stress distribution, crossing point of flange and web and middle point for flange frame have been used.

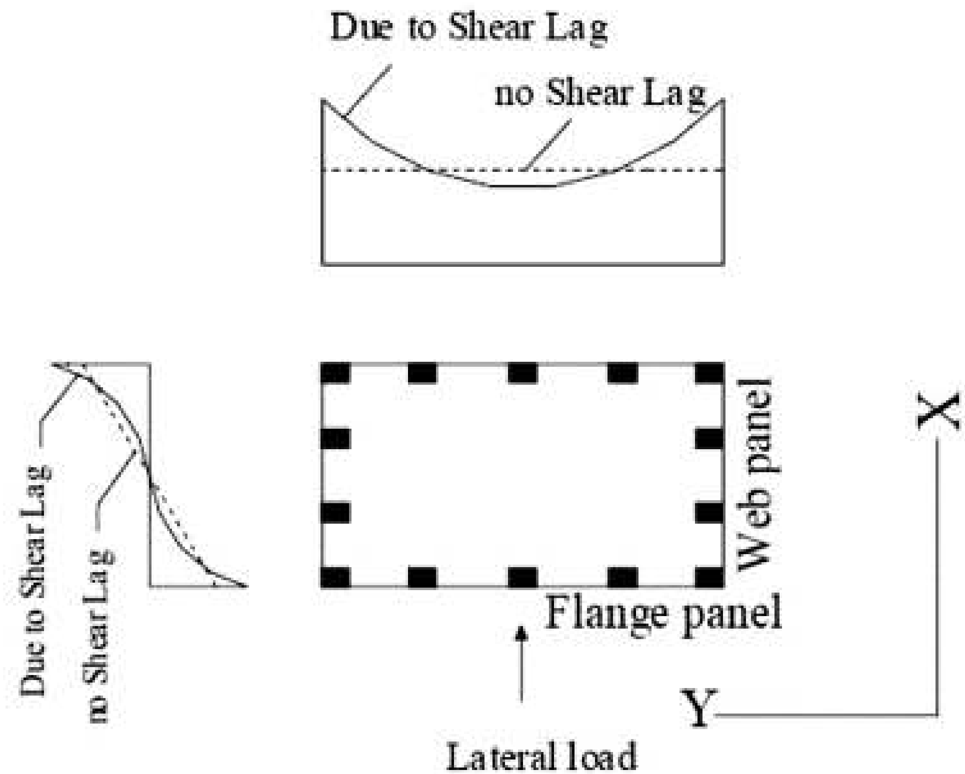

Figure 3: distrution of axial stress in tubular structure

Table 2: λ function for three different mode of loading in flange and web plates

Load type	λ
Concentrated load above structure	$\lambda_{web} = \dfrac{1}{a + bX^2 + \dfrac{c}{X}}$ $\lambda_{flange} = e^{(a+cX)/(1+bX)}$
Monotonous spread load	$\lambda_{web} = a + \dfrac{b}{X} + \dfrac{c}{X^2}$ $\lambda_{flange} = \dfrac{1}{a + bX^2}$
Linear spread load	$\lambda_{web} = a + bX^3 + \dfrac{c}{\sqrt{X}}$ $\lambda_{flange} = a + bX + cX^2 + dX^3 + eX^4 + fX^5$

Table 3: Constant factors of λ functions

Load type	Web Flange	a	b	c	d	e	f
Concentrated	web	0.023	0.25	0.00047	-	-	-
load	flange	0.4	-0.988	-0.364	-	-	-
Monotonous	web	-4176.8	2326.52	-212.62			
Load	flange	1.09	-4.6	-	-	-	-
Linear load	web	0.012	0.11	-0.047	-	-	-
	Flange	2.41	-2.42	-76.32	244.33	-507.28	359.5

In above relations, $X=\frac{z}{h}$. In the equation of web frame, λ is related to end point of web and crossing point of flange and web and stress is equal to zero in the middle point of web. In the equation of flange frame, λ values are related to middle point of flange frame. Stress in the end point of flange frame is the same as stress obtained from equation related to web. So, according to stresses obtained in these points, axial stress diagram of spandrel frame columns can be easily drawn in any section.

Regarding equations (16) and (17), we have

$$\sigma_{Web} = Ea\frac{d\phi}{dz}\left[\left(1-\frac{\alpha}{\lambda}\right)\frac{x}{a}+\frac{\alpha}{\lambda}\left(\frac{x}{a}\right)^3\right] \quad ...(18)$$

$$\sigma_{Flange} = Ea\frac{d\phi}{dz}\left[\left(1-\frac{\beta}{\lambda}\right)+\frac{\beta}{\lambda}\left(\frac{y}{b}\right)^2\right] \quad ...(19)$$

Values of $\frac{d\phi}{dz}$ for concentrated, monotonous spread and linear spread respectively are given by relations (20), (21) and (22) respectively.

$$\frac{d\phi}{dz}=\frac{M}{EI}=\frac{p(h-z)}{EI} \quad ...(20)$$

$$\frac{d\phi}{dz}=\frac{M}{EI}=\frac{p(h-z)^2}{2EI} \quad ...(21)$$

$$\frac{d\phi}{dz}=\frac{M}{EI}=\frac{T(h-z)^2}{6h}\left(\frac{z+2h}{EI}\right) \quad ...(22)$$

Relation (23) gives unknown value of equivalent EI using equations of bending moment equilibrium and axial forces of flange and web plates.

$$EI=\frac{4}{3}Et_w a^3\left(1-\frac{2}{5}\alpha\right)+4Et_f a^2 b\left(1-\frac{2}{3}\beta\right) \quad ...(23)$$

Numerical Studies

In this section, an example is examined with three methods including accurate method, Kwan method in (Wang and Chan, 2006) and proposed relations. The examined building is a concrete structure with 40 stories and its other specifications are as follow: elevation of stories 3m, distance of columns from each other 2.5m, dimension of all beams and columns is equal to 0.8m × 0.8m, monotonous spread load equal to 120 KN/m, E=20Gpa, G=1441 equal to 2a=30m and 2b=35m.

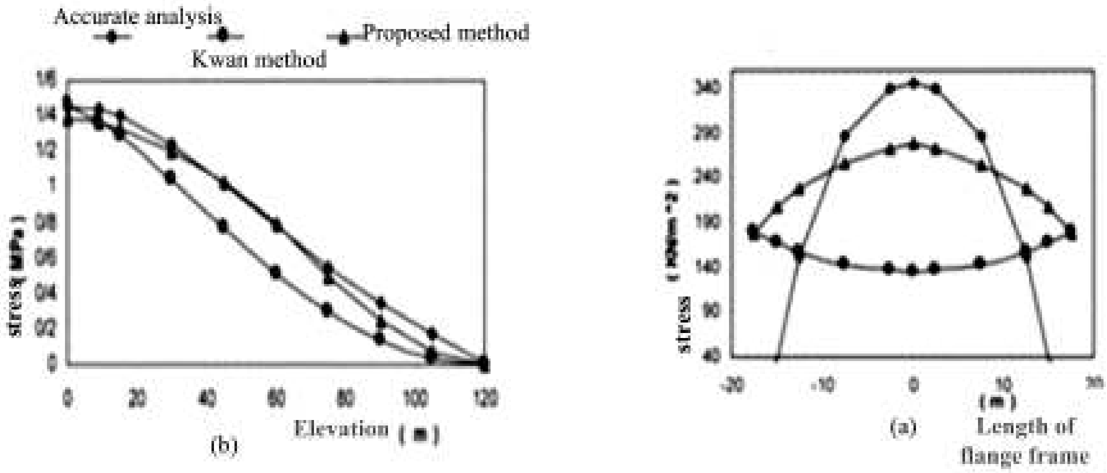

Figure 4 : a) stress in columns of flange frame versus the elevation of 90m, b) stress of middle column of flange versus the elevation of structure

According to figure 4-a, it is indicated that the proposed relations clearly calculate and demonstrate phenomenon of negative shear lag in the above of structure. As evident in figure 4-b, the proposed relations is of good accuracy in total elevation of structure and results of these relation is closer to the accurate analysis than the Kwan method (Wang and Chan, 2006).

CONCLUSION

The proposed method in this paper can describe distribution situation of stresses in frame with fair accuracy that is reliable for various spandrel frames. Regarding above relations, stress in each cells can be obtained using their coordinates. Presented values for examination of positive shear lag in the bottom of structure is of good accuracy for both flange and web frame and its error are equal to 8-15 percent. Investigation of various models for structure elevation reveals that above-mentioned relations account fairly the negative shear lag above structure and of higher accuracy than other references. Drawing diagram of α and β against inertial moment of beam indicate that with increment of inertial moment of beam, effect of positive shear lag decreases. However, slight slope of curvature indicates that sheer increase of inertial moment of beam, isn't an economic solution to decrease effects of shear lag. Drawing diagram of α and β against cross section of columns indicates that the effects of shear lag in whole system decreases phenomenally with increment of column's cross section and the slope of curvature indicates that increment of column's cross section has better effect on shear lag than increment of inertial moment of beam. Drawing stress diagram in flange frame for various models against structure elevation and investigation the effect of changing various parameters on critical point of shear lag reveals that variations of cross sections of columns have maximum effect on the displacement of critical point of shear lag. The critical point of shear lag is located in the distance of 0.25 to 0.33 of structure elevation.

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