



FLUID MOVEMENT AND THERMAL ENERGY MECHANISMS IN A PLATE HEAT TRANSFER: A MATHEMATICAL MODEL

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ABSTRACT

The elliptical and hyperbolic mathematical models based on boundary conditions are usually unfamiliar in a closed-form in the experimental measurements of the differential equation that resolve fluid and heat transesterification. To obtain approximate results of an equation system, the boundary condition of equations is needed. When a system of equations is used to estimate two-dimensional stationary problems, the most simple two-dimensional method of estimation is recommended.

KEYWORDS: Mathematical Model, Fluid Flow, Heat Transfer, Schematic Diagram

When it comes to energy and process engineering investigation techniques, the most common approach used is models and modelling.

Of course, commonly used equations like formulation, differentiation, and other empirical computations are used in mathematical modelling (Stavreva *et al.*, 2013) (Dimitrieska *et al.*, 2015).

The experimental findings and the explanations for their derivation from these equations will be the subject of this article. The goal of this article is not to go into great detail about these mathematical equations, and they are only included if necessary. Based on prior knowledge, these formulas and their formulations are assumed to be related and quite well.

The Poisson approximation for flow direction – spiral, (Ransau, 2002) (Lapidus and Pinder, 1982) (Staffan Grundberg, 2002), and the elliptical simulation parameters in the simple two-dimensional structure are suggested when structures of formulas are applied to technical challenges, particularly for two rectangular fluid – process simulation in a heat plate heat transfer.

The substitution of vortex velocity components and streamline velocity components are needed in this method of approaching the problem.

The vortex's vector is described as follows: (Ransau, 2002):

$$\omega = \Delta^* u \dots\dots\dots (1)$$

SUGGESTED SOLUTION

Using the Cauchy - Riemann velocity potential conditions, (Ransau, 2002):

$$\frac{\partial \psi}{\partial x} = -v$$

$$\frac{\partial \psi}{\partial y} = u \quad (2)$$

The following parameters are solved for two-dimensional persistent liquid dynamics, independent of the existence of fluid, using the two-dimensional boundary condition for scalar vortex and streamline:

$$u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} = v \left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) \quad (3)$$

The well-known vortex-transport equation, (3), is a parabolic differential equation.

The streamline equation is obtained by ordering (2) and (3):

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega \quad \Delta \psi = -\omega \quad (4)$$

As a consequence of variable substitution, the euclidean incompressible formulas are divided into one linear polarization and one elliptic equation (which is a common problem). The elliptic-parabolic viscous fluid equations are then split into one linear polarization and one elliptic formula. Several of these formulas are calculated using the following method:

1. Ordering of initial values for ω and ψ ;

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2. Calculation of vaertex-transport equation;
3. Calculation of Poission’s Equation for ψ in all points, by using of new values for ω in interior points.
4. Determination of boundary value of ω by using of values for ω and ψ in interior points.
5. Back to step 2 if solution has no convergeable.
6. Determination of velocity component values, u and v.

Idem means obtaining pressure and temperature distribution values, as well as estimated displacement values in the x and y directions, in the defined area, under known boundary conditions, in preparation for the next calculation.

Using this type of problem formulation, there are two methods for solving machine equations. The Gauss procedure of exclusion and the Cramer law are examples of direct methods that have a huge proportion of simple mathematical procedures and a long computational cost. Iterative methods, on the other hand, are easier to program because the calculations are quicker and more reliable (Dimitrieska *et al.*, 2015), (Ransau, 2002).

A type of iterative method is the Jacobi recommended methodology, also known as the point proposed process. Point Seidel, linear incremental method Line Siedel, and ADI methodology are known evolutionary algorithms for an analytical scheme of the framework of elliptical fractional-order and measurement of the stream - velocity potentials, as per (Lapidus and Pinder, 1982) (Staffan Grundberg, 2002). (Alternating Direct Implicit Technique).

The ADI approach was used to estimate the area of streaming under specified boundary conditions, which is more efficient and appropriate for perpendicular cross-sectional channels with given contour.

The approximation of a framework of elliptical formulas is similar to an approximation of a system of parabolic equations when using the ADI approach to solve two-dimensional problems. The Poisson equation is estimated using a simulation of the second-order mean velocity derivation using the finite differences method:

$$\frac{\partial^2 \psi}{\partial x^2} \approx \frac{\psi_{i-1,j} - 2\psi_{i,j} + \psi_{i+1,j}}{(\Delta x)^2} \tag{5}$$

$$\frac{\partial^2 \psi}{\partial y^2} \approx \frac{\psi_{i,j-1} - 2\psi_{i,j} + \psi_{i,j+1}}{(\Delta y)^2}$$

As per the condition $y=x$, the final form of the Poisson equation is related by summarizing formulas in (5) and identification with (4):

$$\psi_{i,j} = \frac{\psi_{i-1,j} + \psi_{i,j-1} + \psi_{i+1,j} + \psi_{i,j+1} + \omega_{i,j} \Delta x \Delta y}{4} \tag{6}$$

As a consequence, the streamline values in node I j are influenced by the strength of neighbouring board nodes in the node's area, as well as the values of the node's vortex, as shown in Figure 1. The problem is solved using the TDMA method (Tridiagonal Matrix Algorithm Method) and the Gauss-Seidel iteration.

The equation (6) is simplified in the TDMA form to:

$$\psi_{i,j-1} - 4\psi_{i,j} + \psi_{i,j+1} = -\psi_{i-1,j} - \psi_{i+1,j} - \omega \Delta x \Delta y \tag{7}$$

When the main variable u is introduced, equation (7) takes on the following form:

$$a_i \mathbf{u}_{i-1} + b_i \mathbf{u}_i + c_i \mathbf{u}_{i+1} = d_i \tag{8}$$

Where the coefficients of the finite element differences are a,b,c,d.

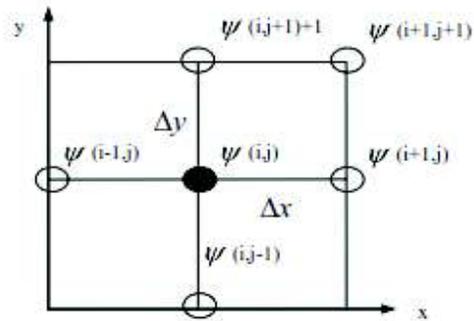


Figure 1: For the Poisson formulation, a scheme of 2D finite differences is shown.

It's similar to the Gauss method of elimination, but it's more precise. As a consequence of the implementation of dependency, (Ransau, 2002):

$$\mathbf{u}_{i-1} = e_i \mathbf{u}_i + f_i \tag{9}$$

Whereupon:

$$e_{i+1} = \frac{-c_i}{a_i e_i + b_i} \quad f_{i+1} = \frac{d_i - a_i f_i}{a_i e_i + b_i} \tag{10}$$

The procedure is reduced to numerical estimation and getting of values for ψ, ω in any interior node in area of interest. Starting from line $i=2$, mention that constant d_i in three agonal equation is composed of known values for $\psi_{i,j}$ and estimated values for $\psi_{3,j}$. By using of TDMA new value $\psi_{2,j}$ is obtained.

Procedure are repeated for $i=3$, by using of values for $\psi_{2,j}$ for assumed of last estimated value for $\psi_{4,j}$. The procedure is repeated step by step, line by line, until the end.

During estimation, criteria for conversation from old (stara) to new(nova) value must be kept, based on defined value of allowed error ϵ .

$$\sum_{i,j} |\psi_{nova} - \psi_{stara}| < \epsilon \tag{11}$$

The amounts of vortex shedding in all domains of the created grid are calculated using the same analogy. However, keep in mind that flow lines and vortex shedding are connected.

TECHNIQUE OF ADI

The ADI method reduces the measurement of the key components, acceleration, and tension, to a simple mathematical measurement of the new mathematical model.

Two equations are used to figure out the velocities:

$$u_{i,j} = \frac{\psi_{i,j+1} - \psi_{i,j-1}}{2\Delta y}$$

$$v_{i,j} = -\frac{\psi_{i+1,j} - \psi_{i-1,j}}{2\Delta x} \tag{12}$$

For the associated importance of initial velocity in the examination module.

The following equations are used to estimate pressure: (Ransau, 2002) (Lapidus and Pinder, 1982) (Staffan Grundberg, 2002):

$$p_{i,j}^{n+1} = \frac{1}{4} (p_{i+1,j}^n + p_{i-1,j}^n + p_{i,j+1}^n + p_{i,j-1}^n) + \frac{1}{4} (2\Delta x^2 \psi(i,j)(d \cdot e - f^2)) \tag{13}$$

Therefore:

$$d = \frac{\psi_{i+1,j} - 2\psi_{i,j} + \psi_{i-1,j}}{\Delta x^2}$$

$$e = \frac{\psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1}}{\Delta y^2}$$

$$f = \frac{\psi_{i+1,j+1} - \psi_{i+1,j-1} - \psi_{i-1,j+1} + \psi_{i-1,j-1}}{4\Delta x \Delta y} \tag{14}$$

For a specified reference level of vertical stress and a given temperature t .

Exact knowledge of drag coefficient in the x and y directions in any node of the created grid is needed for approximation of the ecological formula for two-dimensional temperature distributions. The environmental equation is a logarithmic mathematical expression with a well-known approximation (Ransau, 2002) and (NASA, 2005).

Observations of local convective thermal conductivity, local flow values, and local Rearrange in any network of the generated grid are connected to two-dimensional thermal domains. After obtaining thermophysical characteristics for known input fluid temperature and within schedules, the Pr number, Nu number, and convective coefficient are determined (both temperatures are measurable). For forced disorderly incompressible viscous heated fluid (20-300 0 C) in a triangular cross-section stream between two parallel plates, there are a variety of empirical and semi-empirical associations (Stavreva *et al.*, 2013) (Ransau, 2002) but the following are used in this paper:

$$Re(i,j) = \frac{w(i,j)\delta}{\nu(t_0)} \tag{15}$$

Therefore $w(i,j)$ is a resultant local velocity of the fluid in any node, Δ is characteristic destination between parallel plates, $\nu(t_0)$ is kinematic fluid viscosity for initial temperature t_0 .

$$Nu(i, j) = 0.021 \varepsilon_l \operatorname{Re}(i, j)^{0.8} \operatorname{Pr}(t_o)^{0.43} \left(\frac{\operatorname{Pr}(t_o)}{\operatorname{Pr}_z(t_{oz})} \right)^{0.25} \quad (16)$$

Therefore ξ_1 is correction related with channel geometry and Re number, $\operatorname{Pr}(t_o)$ for input temperature t_o , $\operatorname{Pr}(t_{oz})$ for fluid temperature from interior and exterior side of the plate. Therefore for known thermo physics characteristics of the fluid for wall temperature, $\operatorname{Pr}(t_{oz})$ number is defined in compliance with The coefficient of local heat transfer is calculated as follows:

$$\alpha(i, j) = 0.021 \varepsilon_l \operatorname{Re}(i, j)^{0.8} \operatorname{Pr}(t_o)^{0.43} \cdot \left(\frac{\operatorname{Pr}(t_o)}{\operatorname{Pr}_z(t_{oz})} \right)^{0.25} \frac{\lambda(t_o)}{\delta} \quad (18)$$

Continuous heat transfer flux is rated for known observable quantities of fluid and plate concentration on the internal and external parts of the structure, as well as established hardness and heat absorption of the machining process.

$$q = \alpha_v(i, j)(t_v(i, j) - t_z^v) = \text{const}$$

$$q = \frac{\lambda_m}{\delta_m} (t_z^v - t_z^n) = \text{const} \quad (19)$$

$$q = \alpha_n(i, j)(t_z^n - t_n(i, j)) = \text{const}$$

The direction of energy transfer is:

$$t_v(i, j) > t_z^v > t_z^n > t_n(i, j) \quad (20)$$

CONCLUSION

The computer analysis that has been debunked is in statistical values that can be used to solve a program. For known velocity field and pressure values, local climate data in any node of the grid are calculated using a known technique. For mean temperature in the inlet, internal, and responsibility of the executive of the module worked like a heat plate exchanger and for wall temperatures from the other sides, the temperature distribution in nodes is calculated for given cultural cultures of heat transfer coefficients, under known physicochemical fluid morphology and metal sheets. The process is repeated until the climate models are correct enough to be useful. The corresponding dates and from both sections of the plate evaporator may be used to estimate convective heat transfer. This model can be

modified to suit a pipe wall or a plate. The material of the pipe or plate, as well as the conductivity coefficient, can be used to calculate the thermal flux.

The key thermohydraulic processes can be achieved by integrating the conjectured computational equations in two dimensions. It is the foundation for all three-dimensional equations. The key parameters that describe fluid flow processes are connected in this way.

REFERENCES

- Лыков А.В., 1978. Теплообмен – Справочник, Энергия – Москва.
- Bejan A., 1993. Advanced Engineering Mathematics, Seventh Edition, John Wiley & Sons, INC. S.
- Ransau R., 2002. Solution Methods for Incompressible Viscous Free Surface Flows A Literature Review; Preprint Numeric No.3/2002, Norwegian University of Science and Technology, Trondheim, Norway.
- Computational Fluid Dynamics, NASA Langley Research Center in Hampton, V.A.; Fluid Mechanics and Acoustics Division, 2005, USA.
- Lapidus L. and Pinder G.F., 1982. Numerical Solution of Partial Differential Equations in Science and Engineering, Princeton University, John Wiley & Sons.
- Staffan Grundberg, 2002. Numerical Solution of Navier-Stokes equation using FEMLAB, Umea University, Department of Physics.
- Stavreva S., Serafimov M., Andreevski I. and Dimitrieska C., 2013. Used of CFD Analysis to Achieve Energy-Efficient Data Center, 12th Anniversary International conference on accomplishments in Electrical and Mechanical Engineering and Information Technology, DEMI 2013, Banja Luka, BIH, 31.
- Dimitrieska C., Andreevski I., Vasilevska P.S. and Stavreva S., 2015. Numerical Solution of Mathematical Model for Fluid Flow and Heat Transfer Processes in Plate Exchanger base on Streamline - Vortex Method, 12th international conference on accomplishments in electrical and mechanical engineering and information technology, DEMI 2015, Banja Luka, BIH.