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URBAN POPULATION GROWTH: A LINEAR ALGEBRAIC APPROACH

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ABSTRACT

Cities worldwide are growing at an unprecedented rate, leading to significant changes in population distribution. Understanding these trends is crucial for analyzing urban development and predicting future growth. In this study, we apply Linear Algebra to model and analyze population changes using vector spaces, matrix transformations, and eigenvalue analysis. With real-world data from cities such as New York, Tokyo, and Delhi, we constructed a migration matrix *M* to track how population changes between different regions over time. By studying the properties of *M*, we can identify stable population distributions and long-term demographic trends. To implement our models, we use Python, leveraging NumPy, Pandas, and Matplotlib for computations and visualization. By analyzing successive powers of the migration matrix, we estimate future population distributions and assess the extent of urban expansion. This approach allows us to study key factors that influence urban population trends, such as migration rates, birth-to-death ratios, and economic changes. As physics students, we approach this study from a systems and modeling perspective, focusing on how mathematical structures can describe real-world dynamics. By combining Linear Algebra with computational methods, we provide a structured approach to understanding urban population change, demonstrating how physics-based modeling techniques can be applied to large-scale societal problems.

KEYWORDS: Population Distribution, Python, Computational Methods, Linear Algebra

Population growth is one of the most pressing challenges faced by India today. With each passing year, the country's population reaches new heights, placing significant demands on infrastructure, resources, and economic planning. In April 2023, India officially surpassed China to become the most populous country in the world—an astonishing fact, considering that China's land area is nearly 2.9 times larger than India's. This rapid population increase raises concerns about sustainability, resource distribution, and urban planning. It can be analyzed by studying the demographic and spatial dimensions of the Delhi metropolitan dynamics in terms of population growth (including migration dynamics), distribution (and redistribution), and spatial expansion (emergence of urban peri-urban areas) (Kumar and Lal, 2023; Hamdy *et al.*, 2016). Mathematical models such as the Exponential and Logistic growth models were applied to model the population growth of India using the Census data of India. The exponential model predicted a growth rate of 2. 3% per year and predicted the population until 2050 (Dupont, 2004). Urban growth, driven by population expansion, is analyzed using satellite data and hybrid Cellular Automata (CA) models such as CA-Markov Chain (CA-MC), CA- Logistic Regression (CA-LR) and CA-MC-LR. These models dominate geospatial analysis, generating satellite-based Land Use Land Cover (LULC) maps to study land cover and demographic changes. By integrating population growth trends, calibrated models improve predictions of future urban expansion (Tripathy and Kumar, 2019). Our approach relies on using Markov Chains with Linear Algebra and computational methods, which provide a structured way to analyze how populations shift over time. Using migration matrices and Python-based modeling, we can simulate future trends in Delhi-NCR up to 2030 using the data provided by CENSUS of India.

Our approach relies on Linear Algebra and computational methods, which provide a structured way to analyze how populations shift over time. Using migration matrices and Python-based modeling, we can simulate future trends and gain insight into how urbanization is shaping India's cities. Through this study, our aim is to show how math can be used in real-world applications and how population modeling can help in long-term urban planning.

Objective of the Paper

In this paper, we will walk you through a simple yet effective way to estimate the population for the upcoming years using Linear Algebra and real-world data. Our method combines existing census data with mathematical models, providing an intuitive approach to predicting population trends.

We have two main objectives:

1. Understanding Population as a Mathematical

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Structure: We will show how we can represent the population as vector variables and apply Linear Algebra techniques to analyze them. By constructing a migration matrix, we can model how people move between different regions and predict how the population will change over time. This will help us understand the long-term trends and stability of urban growth.

2. Using Python to Solve These Equations Practically: Once we have established the mathematical foundation, we will demonstrate how to implement these equations using Python. We will use basic libraries such as NumPy and SciPy to perform matrix operations and visualize our results with Matplotlib. This will show how mathematical modeling can be applied in real-world scenarios with just a few lines of code.

Mathematical Structure

We will use Markov chains as an application of Linear Algebra. A general non-homogeneous differential equation is:

$$y_{k+n} + a_1 y_{k+1n-1} + \ldots + a_n y_k = Z_\mu; \quad (1)$$

The solution for the above equation can be given as a difference equation, i.e.

$$y_{k+1} = Ay_k, \qquad \text{for all k} \quad (2)$$

where y_k are vectors in R^n and A is an nxn matrix.

Now we will assume the population of cities as nonnegative probability vectors. The matrix of these probability vectors is called the stochastic matrix. A Markov chain is then defined as a sequence of probability vectors $x_0, x_1, x_2, \ldots$ together with a Stochastic Matrix M, such that

$$x_1 = Px_0, x_2 = Px_1, x_3 = Px_2, \ldots \quad (3)$$

Thus, the Markov chain is expressed by the first-order difference equation

$$x_{k+1} = Px_k, \qquad for \qquad k = 0, 1, 2, \ldots \quad (4)$$

Here, x_k is called a state vector as it describes the 'current state' of the system according to the Stochastic Matrix P. It can vary as it is directly proportional to Probability Matrix.

Example

We took the example of Delhi to illustrate the large flow of people from different parts of the country. Due to this, the population of Delhi has grown exponentially as illustrated by [2] since 1951. However, the population rise in the past ten years can be approximated by a linear model. We have exploited this linear growth of the population to obtain predictions for the population of Delhi over the next five years. (Table 1)

The structure of the migration matrix M is the following:

$$M = \begin{matrix} \text{Immobile Delhi Population} & \text{Migratory Population coming to Delhi} \\ \text{Migratory Population going out from Delhi} & \text{Immobile Outside Population} \end{matrix} \quad (5)$$

The migration matrix is multiplied by the 2x1 vector V:

$$V = \begin{matrix} \text{Current Population of Delhi} \\ \text{Current Population of Outside States} \end{matrix} \quad (6)$$

The result of this multiplication is a 2x1 result matrix R:

$$R = \begin{matrix} \text{Predicted Population of Delhi next year} \\ \text{Predicted Population of Other States next year} \end{matrix} \quad (7)$$

This procedure can be repeated to obtain population predictions for subsequent years by multiplying M by R. To obtain population prediction for 2030 for example, we took the current population of Delhi and other states and post-multiplied it with our migration matrix M to get the prediction matrix R for 2025. Then we used that matrix as V to find R for 2026 and so on four more times.

Table 1: Year wise Population Details

Year	Delhi Population	Other States Population
2014	25,039,000	356,000,000
2015	25,866,000	370,150,000
2016	26,720,000	376,400,000
2017	27,602,000	384,000,000
2018	28,514,000	390,200,000
2019	29,399,000	396,350,000
2020	30,291,000	402,750,000
2021	31,181,000	405,650,000
2022	32,066,000	409,850,000
2023	32,941,000	418,000,000
2024	33,807,000	426,000,000
2025	34,689,027	433,005,465

Prediction for 2016 using 2015 data:

$$M = \begin{matrix} 0.98 & 0.015 \\ 0.02 & 0.985 \end{matrix} \begin{matrix} 25866000 \\ 370150000 \end{matrix} = \begin{matrix} 30900930 \\ 365115070 \end{matrix} \tag{8}$$

$\%\ Error = 15.65\%$

Prediction for 2017 using 2016 data:

$$M = \begin{matrix} 0.98 & 0.015 \\ 0.02 & 0.985 \end{matrix} \begin{matrix} 27602000 \\ 384000000 \end{matrix} = \begin{matrix} 31831600 \\ 371288400 \end{matrix} \tag{9}$$

$\%\ Error = 15.32\%$

Prediction for 2018 using 2017 data:

$$M = \begin{matrix} 0.98 & 0.015 \\ 0.02 & 0.985 \end{matrix} \begin{matrix} 28514000 \\ 390200000 \end{matrix} = \begin{matrix} 32809960 \\ 378792040 \end{matrix} \tag{10}$$

$\%Error = 15.07\%$

and so on for the next years. To calculate the population for the upcoming year, we are not using the calculated data of the previous year. Instead, we are using the actual data to calculate the upcoming year's population. (Table 2)

Table 2: Upcoming Year's Population

Year	Actual Delhi	Predicted Delhi	Error (%)
2015	25,866,000	29,878,220	15.51%
2016	26,720,000	30,900,930	15.65%
2017	27,602,000	31,831,600	15.32%
2018	28,514,000	32,533,940	15.07%
2019	29,399,000	33,796,720	14.96%
2020	30,291,000	34,756,270	14.74%
2021	31,181,000	35,726,430	14.58%
2022	32,066,000	36,330,320	14.27%
2023	32,941,000	37,572,430	14.06%
2024	33,807,000	38,552,180	14.04%
2025	34,689,027	39,520,860	13.93%

As the percentage error is decreasing, we have assumed a constant error percentage of around 14 percent for the predictions of the upcoming years.

Python Structure

We used the following libraries:

- Pandas: To store and retrieve population data efficiently.
- Numpy: To perform mathematical calculations.
- Matplotlib: To plot our predicted population

Here is the code:

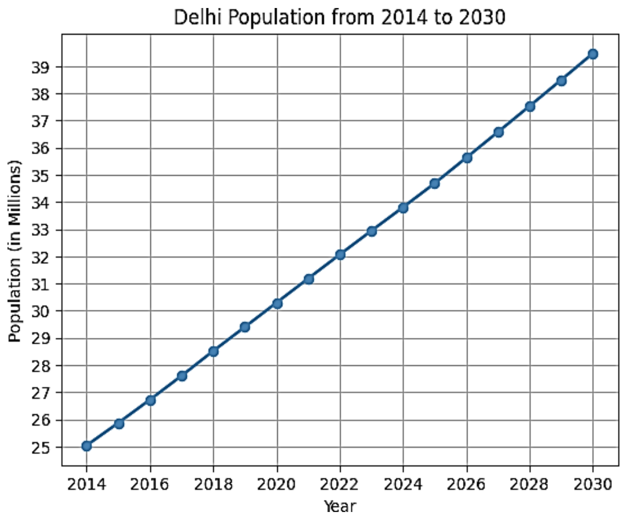
[ ]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

# Define the updated transition matrix
P = np.array ([[0.98 , 0.015] , # Higher retention in Delhi , slightly more
    migration
[0.02 , 0.985 ]]) # Allow some migration away from 'Other States'
# Load population data from CSV
csv_file = "population_data .csv"
df = pd.read_csv (csv_file )

# Extract the latest known actual population as input for prediction
latest_population = np.array ([df.iloc [-1]['Actual Delhi '], df.iloc [-1]['Actual
     Other States ']])
results = []
current_year = int( df.iloc [-1][' Year '])
years_to_predict = 2030 - current_year # Number of years to predict

for i in range (1 , years_to_predict + 1):
    next_year = current_year + i
    predicted_population = P @ latest_population

    # Adjust predicted population with constant error correction
    adjusted_population = predicted_population . copy ()
    adjusted_population [0] -= adjusted_population [0] * 0.14 # Subtract 14%
        from Delhi
```

```
    adjusted_population [1] += adjusted_population [1] * 0.0374 # Add 3% to Other States

    # Store actual values if available
    actual_delhi = df[df['Year'] == next_year]['Actual Delhi'].values
    actual_other_states = df[df['Year'] == next_year]['Actual Other States'].values
    results.append([next_year, int(adjusted_population[0]), int(adjusted_population [1])])

    # Update latest population for next iteration
    latest_population = adjusted_population

# Display results in a tabulated format
print("\nPredicted Population from Next Year till 2030")
print("-----------------------------------------------------------------")
print("Year | Delhi Predicted | Other States Predicted")
print("--------------------------------------------------------")
for row in results:
    print(f"{row[0]:<5} | {row[1]:<15} | {row[2]:<23}")

#Plot the result
pop_data = []
for i in range(12):
    pop_data.append(int(df.iloc[i]['Actual Delhi']))
for j in range(5):
    pop_data.append(results[j][1])

pop_data_millions = [val / 1e6 for val in pop_data]
year = [[2014 +i] for i in range(17)]
plt.plot(year, pop_data_millions, color='steelblue', linewidth=2, marker='o')
plt.title("Delhi Population from 2014 to 2030")
plt.xlabel("Year")
plt.ylabel("Population (in Millions)")
plt.grid(True)
plt.xticks(np.arange(2014, 2031, 2))
plt.yticks(np.arange(int(min(pop_data_millions)), int(max(pop_data_millions) + 1), 1))
plt.show()
```

The predicted result for Delhi and Other States is tabulated below:

Table 3: Proposed Population Data

Year	Proposed Delhi Population	Proposed Other States Population
2025	34,778,356	436,004,841
2026	35,748,118	446,248,332
2027	36,719,655	456,735,652
2028	37,695,941	467,472,161
2029	38,679,612	478,463,401
2030	39,673,014	489,715,088



RESULTS AND DISCUSSION

The population prediction model developed for Delhi and other states has been evaluated based on past data (2014–2025) and further extended to predict population trends until 2030.

Error Analysis

- We observed an average error of 14.63% for Delhi and 3.85% average error for other states from 2014-25. Also, the prediction error for Delhi is decreasing by year.
- The higher prediction error for Delhi suggests that population dynamics in the city are more complex, possibly due to factors such as migration from smaller villages, urbanization, birth- to-death ratio and economic and policy changes.

Population Growth Trend (2014–2025)

- Both actual and predicted data indicate a steady rise in population for both Delhi and other states.
- Delhi's population increased from 25.86 million (2015) to 34.6 million (2025), with the pre-diction model closely following the actual values.
- Other states saw relatively moderate growth, with an average error of 3.78%, indicating that the model is more accurate in predicting trends outside Delhi.

Projected Growth (2026–2030)

- If current trends continue, Delhi's population is expected to reach 39.67 million by 2030, while other states collectively could have a population of 489.7 million.
- The projected trend suggests a near-linear growth pattern, meaning no drastic acceleration or slowdown in population increase within this time frame.

Future Possibilities

- By using more robust data and adjusting the model in integration with CA-MC and Linear Regression models can enhance the prediction accuracy.
- Factors like economic changes, birth/death rate and more states migration data could be included to improve reliability.

CONCLUSION

Our devised model predicts the population up-to some extent but needs more refining and robust struc- ture. Although, having a constant 14% error for Delhi can be incorporated.

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