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AMPLITUDE Nth-POWER SQUEEZING OF RADIATION IN INTERACTION OF TWO TWO-LEVEL ATOMS WITH SINGLE MODE COHERENT RADIATION

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ABSTARCT

We study amplitude nth-power squeezing for the general quadrature amplitude nth-power operator $X_\theta^{(n)} = \frac{1}{2}[a^n e^{-i\theta} + a^{+n} e^{i\theta}]$, in interaction of a single mode coherent radiation with an superradiant two two-level atoms using the Hamiltonian, $H = \omega(a^+a + S_z) + g(aS_+ + a^+S_-)$, in the natural system of units. Here, $\alpha = |\alpha|e^{i\theta_\alpha}$, a and a^+ are annihilation and creation operators for radiation respectively which obey bosonic commutation relation $[a, a^+] = 1$, S_z and $S_\pm$ are the collective Dicke operators, g is the coupling constant, ω is the energy of the photons and the energy difference between the two atomic levels. We solve the interaction problem exactly and obtained amplitude nth-power squeezing in such interaction for an arbitrary power n by choosing suitably the value $|\alpha|$ and coupling time gt. Variations of amplitude nth-power squeezing for some values of n with intensity of radiation and coupling time have also been discussed.

KEYWORDS: Coherent State, Squeezing, Amplitude-Squared Squeezing, Higher-Order Squeezing, Phase Shifting Operator, Dicke Model

The non-classical effects of a state (Loudon and Knight, 1987) can be manifested in different ways like squeezing, anti-bunching and sub-Poissonian photon statistics etc. A number of nonlinear optical systems that may be able to produce non-classical states have been analyzed theoretically. Usefulness of such states has been understood because of its potential applications in quantum information theory such as quantum communication (Bennett *et al.*, 1999), quantum teleportation (Braunstein, 2000), dense coding (Braunstein and Kimble, 2000) and quantum cryptography (Bennett *et al.*, 1992) are well realized. It has been demonstrated that non-classicality is the necessary input for entangled state (Kim, 2002).

Squeezing, a well-known non-classical effect, is a phenomenon in which variance in one of the quadrature components become less than that in vacuum state or coherent state (Glauber, 1963) at the cost of increased fluctuations in the other quadrature component. The concept of ordinary squeezing has been generalized to the 2Nth order (N = 1, 2,….) case by Hong and Mandel (Hong and Mandel, 1985). An alternative generalization of squeezing has been described by Hillery (Hillery, 1986) known as amplitude-squared squeezing. On the basis of Hillery's work, Zhang et al (Zhang *et al*, 1990) introduced amplitude Nth power squeezing, including ordinary squeezing and amplitude-squared squeezing as two special examples.

Consider the most general quadrature amplitude nth -power operator,

$$X_\theta^{(n)} = \frac{1}{2}[a^n e^{-i\theta} + a^{+n} e^{i\theta}]. \tag{1}$$

Commutation relation between $X_\theta^{(n)}$ and $X_{\theta+\frac{\pi}{2}}^{(n)}$,

$$[X_\theta^{(n)}, X_{\theta+\frac{\pi}{2}}^{(n)}] = \frac{i}{2} W^{(n)};$$

$$W^{(n)} = \sum_{r=1}^{n} r!({}^nC_r)^2 a^{+n-r} a^{n-r} \tag{2}$$

and therefore the uncertainty relation between $X_\theta^{(n)}$ and $X_{\theta+\frac{\pi}{2}}^{(n)}$,

$$\left\langle(\Delta X_\theta^{(n)})^2\right\rangle\left\langle(\Delta X_{\theta+\frac{\pi}{2}}^{(n)})^2\right\rangle \geq \frac{1}{16}\left\langle W^{(n)}\right\rangle^2. \tag{3}$$

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A state is said to be amplitude nth-power squeezing in $X_\theta^{(n)}$ if

$$\left\langle (\Delta X_\theta^{(n)})^2 \right\rangle < \frac{1}{4}\left\langle W^{(n)} \right\rangle, \quad (4)$$

and similarly amplitude nth-power squeezing for $X_{\theta+\frac{\pi}{2}}^{(n)}$ can be defined.

Jaynes-Cumming model (JCM) is one of the most frequently used models in quantum optics. Much attention has been devoted to the JCM of field-atom interactions. A large number of studies have shown that this model has a lot of interesting quantum features, such as collapses and revivals and squeezing of radiation. Ordinary squeezing, amplitude-squared squeezing and amplitude Nth power squeezing have been studied. We studied ordinary squeezing and sub-Poissonian statistics of radiation in interaction of two two-level atoms with a single mode coherent radiation (Prakash and Kumar, 2007; Prakash and Kumar, 2010; Kumar and Kumar, 2013; Lynch, 1986; Mandel, 1982; Kim and Yoon, 2002). In the present paper, we study amplitude nth-power squeezing in interaction for atoms initially in superradiant with single mode coherent radiation. Variations of amplitude nth-power squeezing with coupling time, square root of mean photon number and phase have also been discussed.

AMPLITUDE-Nth POWER SQUEEZING OF RADIATION IN INTERACTION OF SUPERRADIANT TWO TWO-LEVEL ATOMS WITH A SINGLE MODE COHERENT RADIATION

Consider a system of two two-level atoms interacting with a single resonant mode of radiation with zero detuning. If the atoms are located in a region small in comparison with the wavelength of the field, but not so small so as to make them interact directly with each other, the Hamiltonian (Dicke, 1954) of the system in the dipole and rotating wave approximation is given in the natural system of units ($\hbar = 1$) by

$$H = H_0 + H_I;\ H_0 = H_F + H_A, \quad (5)$$

Here, $H_F = \omega_F N$, $H_A = \omega_A S_z$, $H_I = g\,(a\,S_+ + a^+ S_-)$, $N = a^+a$ and subscripts F, A, and I refer to field, atoms and interaction, N, a and a^+ are number, annihilation and creation operators respectively, g is coupling constant and $S_{\pm,z}$ are the Dicke's collective atom operators (Dicke, 1954). Since $[H_0, H_I] = 0$, the time evolution operator $U = e^{-iHt}$ can be written as $U = U_0 U_I$, where $U_0 = e^{-iH_0 t}$ and $U_I = e^{-iH_I t}$. The exact time evolution operator in interaction picture in the form (Prakash and Kumar, 2007),

$$U_I = e^{-iH_I t} = \begin{pmatrix} 1 + (N+1)\,C(N+1) & -i\,S(N+1)\,a & C(N+1)\,a^2 \\ -ia^+\,S(N+1) & \cos(gt\sqrt{4N+2}) & -i\,S(N)\,a \\ a^{+2}\,C(N+1) & -ia^+\,S(N) & 1 + N\,C(N-1) \end{pmatrix}. \quad (6)$$

Here $C(N) \equiv \{\cos(gt\sqrt{4N+2}) - 1\}/(2N+1)$ and $S(N) \equiv \{\sin(gt\sqrt{4N+2})\}/\sqrt{2N+1}$.

A single mode coherent state $|\alpha\rangle$ defined $a|\alpha\rangle = \alpha|\alpha\rangle$ and is given by

$$|\alpha\rangle = \exp\left(-\frac{|\alpha|^2}{2}\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle\,. \quad (7)$$

Here $|n\rangle$ is number state defined by $N|n\rangle = n|n\rangle$; $N = a^+a$ .If both atoms are in superradiant state and radiation is in the coherent state $|\alpha\rangle$ initially, i.e., for initial state $|1,0\rangle|\alpha\rangle$, the final state,

$$|\psi\rangle = [-iS(N+1)]\,a\,|1,1\rangle|\alpha\rangle + \cos\sqrt{2(2N+1)}\,gt\,|1,0\rangle|\alpha\rangle + [-i\,a^+\,S(N)]\,|1,-1\rangle|\alpha\rangle\,. \quad (8)$$

Tedious but straight forward calculations lead to results,

$$\langle\psi|a^r|\psi\rangle = A_1 e^{ir\theta_\alpha}\ ,\ \langle\psi|a^{+r}a^r|\psi\rangle = A_2\,. \quad (9)$$

Here

$$A_1 = |\alpha|^2 e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!} \Big[|\alpha|^2 \left[S(n+1)S(n+r+1)\right] + \left[\cos \sqrt{2(2n+1)}\,gt \ \cos \sqrt{2(2n+2r+1)}\,gt \right] + \left[S(n)(n+r+1)S(n+r)\right]\Big], \qquad (10)$$

$$A_2 = e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!} \Big[2|\alpha|^{2(r+1)}\left[S(n+r+1)\right]^2 + |\alpha|^{2r}\left[\cos \sqrt{2(2n+2r+1)}\,gt\right]^2 + r^2 |\alpha|^{2(r-1)}\left[S(n+r-1)\right]^2 + (2r+1)|\alpha|^{2r}\left[S(n+r)\right]^2 \Big]. \qquad (11)$$

Also we have from Eq. (1),

$$\left\langle \left(\Delta X_\theta^{(n)}\right)^2 \right\rangle = \frac{1}{4}\left[\begin{array}{l} \left(\left\langle a^{2n}\right\rangle - \left\langle a^n\right\rangle^2\right)e^{-2i\theta} + \left(\left\langle a^{+2n}\right\rangle - \left\langle a^{+n}\right\rangle^2\right)e^{2i\theta} + 2\left(\left\langle a^n a^{+n}\right\rangle - \left\langle a^{+n}\right\rangle\left\langle a^n\right\rangle\right) \\ + \sum_{r=1}^{n} r!\left({}^n c_r\right)^2 \left\langle a^{+n-r} a^{n-r}\right\rangle \end{array}\right]. \qquad (12)$$

We can study different orders of amplitude nth-order squeezing by substituting n using Eqs. (9) - (12).

RESULTS AND DISCUSSION

We have used these relations to study amplitude-nth-power squeezing in interaction of coherent radiation with two two-level atoms. In particular we have used C++ programming to find minimum of $\left\langle (\Delta X_\theta^{(n)})^2 \right\rangle$ for n = 2, 4 and 6. We obtained minimum values 0.588889 at gt = 1.41 , $|\alpha|$ = 1.01 and $\theta + \theta\alpha = 0$ or π for n = 2, 0.290057, at gt = 53 , $|\alpha|$ = 23.4 and $\theta + \theta_\alpha = 0$ or π for n=4, and 0.34772 , at gt =36, , $|\alpha|$ = 24.1 and $\theta + \theta_\alpha = 0$ or π for n = 6 respectively..This gives 41%, 69%, 65% squeezing for n = 2, 4, 6… respectively.

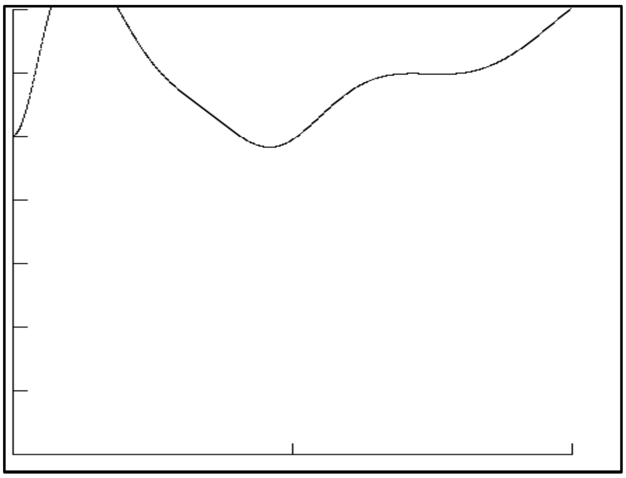

Figure 1: Variation of Amplitude nth-power squeezing factor (n = 2) (on Y axis, 0 to 1.4) with coupling constant gt (on X axis, 0 to 2) for $|\alpha|$ = 1.01, $\theta + \theta_\alpha$= 0 or π

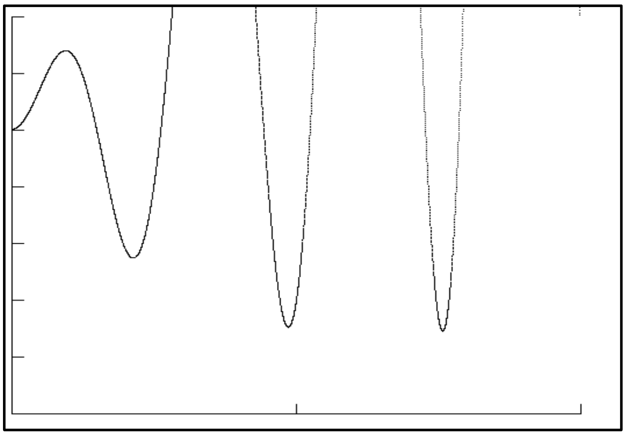

Figure 2: Variation of Amplitude nth-power squeezing factor (n = 4) (on Y axis, 0 to 1.4) with coupling constant gt (on X axis, 0 to 70) for $|\alpha|$ = 23.4, $\theta + \theta_\alpha$= 0 or π

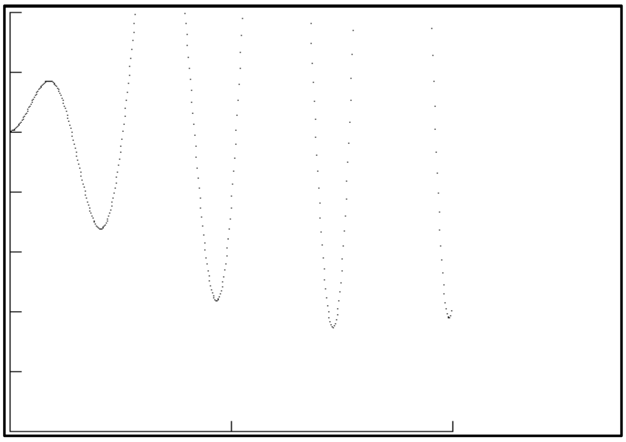

Figure 3: Variation of Amplitude nth-power squeezing factor (n = 6) (on Y axis, 0 to 1.4) with coupling constant gt (on X axis, 0 to 50) for $|\alpha|$ = 24.4, $\theta + \theta_\alpha$= 0 or π

Variations of amplitude nth-order squeezing (for n = 2, 4 and 6) with coupling time gt have been discussed and shown in Figure 1, 2 and 3 respectively. Hence we finally conclude that Amplitude higher order squeezing can be obtained by choosing suitably coupling time and square root of mean photon number.

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