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SQUEEZING OF INFORMATION ENTROPY IN INTERACTION OF TWO TWO-LEVEL ATOMS WITH SINGLE MODE COHERENT RADIATION

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ABSTRACT

In the present paper, we study the information entropy squeezing for two excited two-level atoms interacting with a single mode coherent radiation. We found that information entropy components are alternatively squeezed with the change in noise by phase angle π/2. Variations of information entropy squeezing with different parameters have also been discussed.

INTRODUCTION

Squeezing is a non-classical effect (Walls, 1983; Loudan and Knight 1987), in which the uncertainty in one of the quadrature component is less than that of coherent state at the expense of increased uncertainty in other quadrature component. Its applications (Caves 1981, Loudan 1981) in secure optical communication like quantum key distribution, quantum teleportation, and quantum error coding and gravitational wave detection have been well realized. In last few decades the atomic squeezing has motivated due to its application in atomic clock (Sorenson et al 1998), high resolution spectroscopy, generation of squeezed light (Walls and Zoller, 1981) etc. (Agarwal and Puri, 1990; Wineland et al.1992) and several authors (Kitagawa and Ueda , 1993; Sorensen et al, 2002;Prakash and Kumar, 2005) have been studied about atomic squeezing in different system. All these studies of atomic squeezing are based on Heisenberg uncertainty relation. It may be noted that the definition of atomic squeezing, based on the Heisenberg uncertainty principle, is not sufficient to give information about atomic squeezing in some special cases, which can be confronted by an alternative definition of squeezing (Fang et al, 2000), based on information entropy theory, which overcomes the disadvantages of the definition based on the Heisenberg uncertainty relation. The entropy squeezing of the atom has been studied extensively (Yan et al, 2009; Kayhan 2011). The entropy squeezing based on information entropy theory is more precise than their variance squeezing based on Heisenberg uncertainty relation. The random phase noise influences the dipole of the interaction. In the case of the interaction with the random phase telegraph noise, the coupling coefficient is modified as (Fang et al, 2000) $g(t) = g_0 e^{-i\phi(t)}$ where g_0 is the non-noisy coupling coefficient and $\phi(t)$ represents the random telegraph which fluctuates between two states of the noise.

We studied entropy squeezing in interaction of two two-level atoms with a single mode coherent radiation. In the present paper, we study entropy squeezing in interaction for atoms initially in excited state with single mode coherent radiation. Variations of entropy squeezing with different parameters have also been discussed. We show that the entropy squeezing is very sensitive to the noise. It disappears in time quickly due to the strongly destructive effect of the noise.

ENTROPY SQUEEZING IN INTERACTION OF TWO TWO-LEVEL ATOMS WITH SINGLE MODE COHERENT RADIATION

Consider a system of two two-level atoms interacting with a single resonant mode of radiation with zero detuning. If the atoms are located in a region small in comparison with the wavelength of the field, but not so small so as to make them interact directly with each other, the Hamiltonian (Dicke, 1954) of the system in the dipole and rotating wave approximation is given in the natural system of units ($\hbar = 1$) by

$$H = H_0 + H_I;\ H_0 = H_F + H_A, \qquad (1)$$

Here, $H_F = \omega_F N$, $H_A = \omega_A S_z$, $H_I = g\,(a\,S_+ + a^+ S_-)$, $N = a^+a$ and subscripts F, A, and I refer to field, atoms and interaction, N, a and a^+ are number, annihilation and creation operators respectively, g is coupling constant and $S_{\pm,z}$ are the Dicke's collective atom operators (Dicke, 1954). Since $[H_0, H_I] = 0$, the time evolution operator $U = e^{-iHt}$ can be written as $U = U_0 U_I$, where $U_0 = e^{-iH_0 t}$ and $U_I = e^{-iH_I t}$. The exact time evolution operator in interaction picture in the form (Prakash and Kumar, 2007),

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$$U_I = e^{-iH_I t} = \begin{pmatrix} 1+(N+1)\,C(N+1) & -i\,S(N+1)\,a & C(N+1)\,a^2 \\ -ia^+\,S(N+1) & \cos(gt\sqrt{4N+2}\,) & -i\,S(N)\,a \\ a^{+2}\,C(N+1) & -ia^+\,S(N) & 1+N\,C(N-1) \end{pmatrix}. \tag{2}$$

Here $C(N) \equiv \{\cos(\,gt\sqrt{4N+2}\,)-1\}/(2N+1)$ and $S(N) \equiv \{\sin(gt\sqrt{4N+2}\,)\}/\sqrt{2N+1}$.

A single mode coherent state $|\alpha\rangle$ defined $a|\alpha\rangle = \alpha|\alpha\rangle$ and is given by

$$|\alpha\rangle = \exp(-\frac{|\alpha|^2}{2}) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}|n\rangle . \tag{3}$$

Here $|n\rangle$ is number state defined by $N|n\rangle = n|n\rangle$; $N = a^+a$.

Entropy Squeezing

The fluctuation in the components of spins are squeezed if

$$V(S_k) = \Delta S_k - \sqrt{\frac{|\langle S_z\rangle|}{2}} < 0 \text{ ; } k = x, y.$$

This definition is not appropriate to define the variance squeezing when $\langle S_z\rangle = 0$. The fluctuation in the components of S_α of the atomic dipole are said to be squeezed in entropy (Fang et al, 2000) if the following condition is satisfied:

$$E(S_\alpha) = \delta\, H(S_\alpha) - 2/\,[\delta\, H(S_z)]^{1/2} < 0; \ \alpha = x,y \tag{4}$$

Where $\delta\, H(S_\alpha)$ =exp[$H(S_\alpha)$] denote the information entropy of the component S_k and

$$H(S_\alpha) = -\sum_{i=1}^{N} P_{i(S_\alpha)} \ln P_{i(S_\alpha)} \qquad \alpha = x, y, z$$

where $P_i(S_k)$ represents the probability distribution of N possible measurement of S_k components.

If both atoms are excited and radiation is in the coherent state $|\alpha\rangle$ initially, the initial state is $|\alpha\rangle|1,1\rangle$, and the final state is then obtained using Eq. (2) in the form,

$$|\psi\rangle = [1+(N+1)\,C(N+1)]|\alpha\rangle|1,1\rangle - ia^+\,S(N+1)|\alpha\rangle|1,0\rangle + a^{+2}\,C(N+1)|\alpha\rangle|1,-1\rangle . \tag{5}$$

The Dicke system is composed of three energy levels $|1,1\rangle = |e_1e_2\rangle$, $|1,0\rangle = (|e_1g_2\rangle + |e_2g_1\rangle)/\sqrt{2}$ and $|1,-1\rangle = |g_1g_2\rangle$. These energy levels are known as the Dicke collective states. The states $|1,1\rangle$ and $|1,-1\rangle$ are the excited $|e\rangle$ and ground state $|g\rangle$ levels of individual atoms whereas $|1,0\rangle$ is a superposition of $|e\rangle$ and $|g\rangle$ state of individual atom. In order to analyze the relation between entanglement and spin squeezing parameter, we express the density matrix, $\rho = |\psi\rangle\langle\psi|$ of the system. Using Eq.(5), we find the density matrix of the system in basis of the product states $\{|e_1e_2\rangle, |e_1g_2\rangle, |e_2g_1\rangle, |g_1g_2\rangle\}$ which takes the form,

$$\rho = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{21} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} . \quad (6)$$

Here,

$$m_{11} = e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!}[1+(n+1)C(n+1)]^2, m_{12} = -i\alpha^* e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!}[1+(n+2)C(n+2)]S(n+1),$$

$$m_{13} = m_{12},\ m_{14} = \alpha^2 e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!}[1+(n+3)C(n+3)]C(n+1),\ m_{21} = (m_{12})^*,$$

$$m_{22} = \tfrac{1}{2} e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!}(n+1)[S(n+1)]^2 = m_{23} = m_{32} = m_{33},$$

$$m_{24} = \frac{\alpha^*}{\sqrt{2}} e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!}[(n+2)C(n+1)]S(n+2) = m_{23},$$

$m_{31} = (m_{13})^*, m_{41} = (m_{14})^*, m_{42} = (m_{24})^*, m_{43} = (m_{34})^*$ and

$$m_{44} = e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!}(n+1)(n+2)[C(n+1)]^2.$$

The density matrix of first atom is

$$\rho_{A1} = Tr_{A1A2}(\rho) = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix}$$

Where, $\rho_{11} = m_{11} + m_{22}$, $\rho_{12} = m_{13} + m_{24}$, $\rho_{21} = m_{31} + m_{42}$, $_{22} = m_{33} + m_{44}$ (7)

Using the atomic reduced density operator ρ(t) the information entropies of atomic operators S_x,S_y and S_z can be obtained (Fang, 2000) as following:

$H(S_x) = -0.5[1+2Re\rho_{21}(t)]\ln 0.5[1+2Re\rho_{21}(t)]] -0.5[1-2Re\rho_{21}(t))]\ln 0.5[1-2Re\rho_{21}(t)]$ (8)

$H(S_x) = -0.5[1-2Re\rho_{21}(t)]\ln 0.5[1-2Re\rho_{21}(t)]] -0.5[1+2Re\rho_{21}(t))]\ln 0.5[1+2Re\rho_{21}(t)]$ (9)

$H(S_z) = -\rho_{22}\ln[\rho_{22}] - \rho_{11}\ln[\rho_{11}]$ (10)

we have calculated the entropy squeezing using equations (4) and (7)-(10).

CONCLUSION

Using C^{++} programming we have calculated the entropy squeezing. The variation of entropy squeezing with coupling time gt has been shown in Figure(1). From Figure (1), we see that entropy squeezing occurs at very small coupling time. The entropy squeezing disappears with increasing value of gt and then becomes almost constant.

We have shown that the entropy squeezing is very sensitive to the noise. It disappears in time quickly due to the strongly destructive effect of the noise. Also information entropy components are alternatively squeezed with the change in noise by phase angle π/2 as shown in Figure 2.

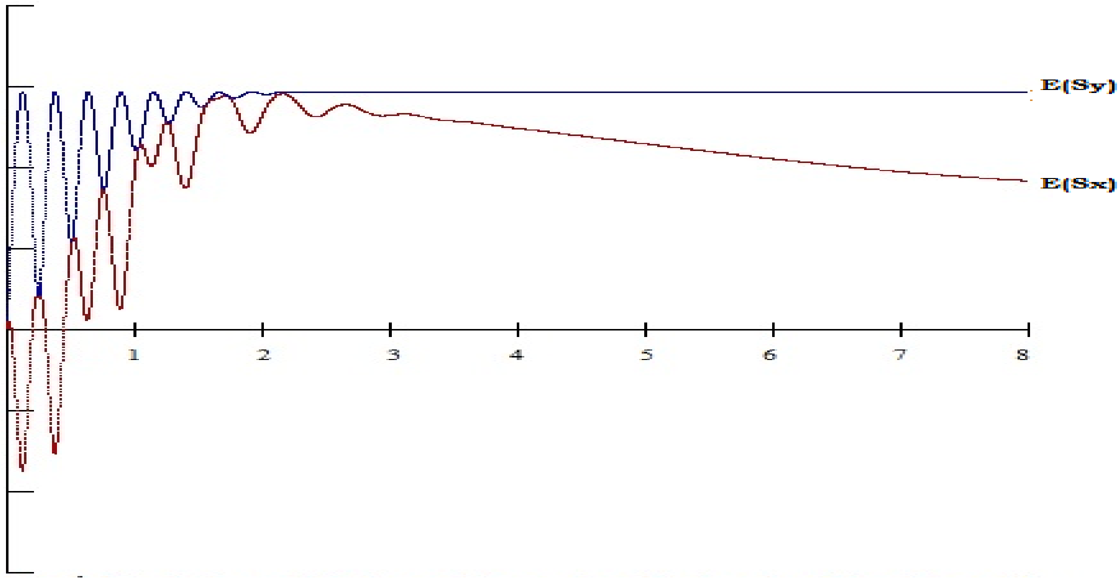

Figure 1: Variation of Entropy Squeezing factor (on Y axis) with coupling constant gt (on X axis) for $|\alpha| = 6$, $\theta + \theta_\alpha = 0$ or π and $\varphi = 0$

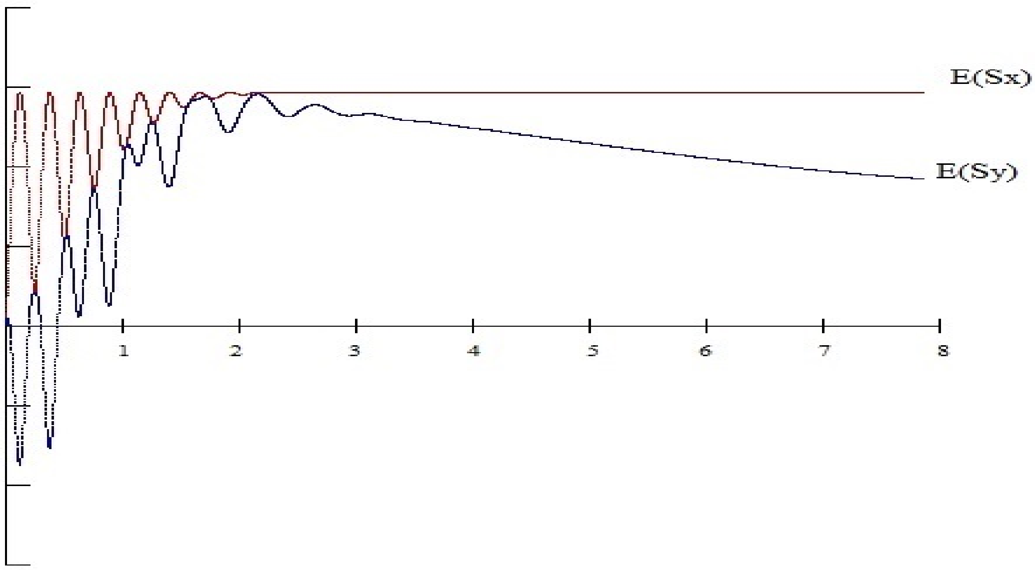

Figure 2: Variation of Entropy Squeezing factor (on Y axis) with coupling constant gt (on X axis) for $|\alpha| = 6$, $\theta + \theta_\alpha = 0$ or π, $\varphi = \pi/2$

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