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SPIN SQUEEZING AND QUANTUM ENTANGLEMENT IN INTERACTION OF TWO EXCITED TWO-LEVEL ATOMS WITH A SINGLE MODE COHERENT RADIATION

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ABSTRACT

In the present paper, we study the relation between spin squeezing and entanglement in two two-level atoms interacting with a single mode coherent field. We use the negativity criterion for the measurement of entanglement and obtain results considering atoms in excited states initially. We conclude that our recently reported criterion for spin squeezing is suitable for characterizing entanglement of composite systems.

KEYWORDS : Coherent State, Two-Level Atom, Spin Squeezing, Quantum Entanglement.

In quantum optics, there are several models which play important roles in the study of interaction of a quantized field with atoms. One of the well known models is the Dicke model (Dicke, 1954) which describes the interaction of a quantized radiation field with a sample of N two-level atoms located within a distance much smaller than the wavelength of the radiation. The simplest case of N = 1 is known under the name of the Jaynes-Cummings model (JCM) (Jaynes and Cummings, 1963). The interaction of a group of two-level atoms with a single mode cavity field has also been considered by Tavis and Cummings (Tavis and Cummings, 1968) and this particular type of Dicke model is termed as the Tavis-Cummings model (TCM). These models lead to several quantum effects such as squeezing of radiation (Dodonov, 2002), amplitude-squared squeezing (Hillery, 1987), Higher-order squeezing (Hong and Mandel, 1985; Prakash et al., 2011) Collapses and Revivals of atomic population (Eberly et al., 1980; Prakash and Kumar, 2008), antibunching or sub-poissonian statistics (Kumar and Prakash, 2008) and spin (atomic) squeezing (Walls and Zoller, 1981; Kitagawa and Ueda, 1993; Wineland et al. 1992,; Sorenson et al., 2002; Prakash and Kumar, 2005).

Squeezing, a well-known non-classical effect, is a phenomenon in which variance in one of the quadrature components become less than that in vacuum state or coherent state (Glauber, 1963) of radiation field at the cost of increased fluctuations in the other quadrature component. In similar analogy to squeezing of radiation field, squeezing of spin components has been defined by several authors (Kumar and Prakash, 2008) and spin (atomic) squeezing (Walls and Zoller, 1981; Kitagawa and Ueda, 1993; Wineland et al. 1992; Sorenson et al. 2002; Prakash and Kumar, 2005). The earliest definition of spin squeezing is according to Walls and Zoller, 1981 who considered commutation relations $[S_x, S_y] = i S_z$ and uncertainty relation, $\left\langle (\Delta S_x)^2 \right\rangle \left\langle (\Delta S_y)^2 \right\rangle \geq \frac{1}{2} \left|\left\langle S_z \right\rangle\right|^2$ and wrote the conditions for squeezing in component in the form,

$$\left\langle (\Delta S_{x \text{ or } y})^2 \right\rangle < \frac{1}{2}\left|\left\langle S_z \right\rangle\right| \quad (1)$$

Later several other authors gave alternate definitions of spin squeezing in the term of squeezing parameter or factor and defined the condition for a state to be spin squeezed as $0 < \xi < 1$. Wineland et al., 1992), defined spin squeezing for spin components in a plane normal to mean spin vector and wrote the spin squeezing factor as,

$$\xi = (2S)\left\langle (\Delta S_{n_i})^2 \right\rangle_{\perp} / \left|\left\langle S_{n_3} \right\rangle\right|, \quad (2)$$

where $\left\langle (\Delta S_{n_i})^2 \right\rangle_{\perp}$ denotes the smallest uncertainty of a spin component perpendicular to mean spin vector, S is the total spin of the system n_1, n_2 and n_3 are the three mutually perpendicular unit vectors oriented such that the mean value of one spin components assumed $\left\langle S_{n_3} \right\rangle$ different from zero, while the other components $\left\langle S_{n_1} \right\rangle$ and $\left\langle S_{n_2} \right\rangle$ have zero mean values. This gives reduction in the frequency noise in the context of Raman spectroscopy. Kitagawa and Ueda, 1993 also defined squeezing of spin components normal to mean spin and wrote the squeezing parameter as,

$$\xi = 2\left\langle (\Delta S_{n_i})^2 \right\rangle_{\perp} / S \quad (3)$$

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Sorensen et al., 2002 proposed the parameter for defining the spin squeezing,

$$\xi = N\left\langle(\Delta S_x)^2\right\rangle / \left\langle S_y\right\rangle^2 + \left\langle S_z\right\rangle^2 \tag{4}$$

Recently Prakash and Kumar, 2005 defined the squeezing parameter for components in the xy-plane as,

$$S_\theta = 2\left\langle(\Delta S_\theta)^2\right\rangle / \left[\left\langle S_{\theta+\pi/2}\right\rangle^2 + \left\langle S_z\right\rangle^2\right]^{1/2} \tag{5}$$

which is a natural generalization of the earliest and simplest definition of Walls and Zoller, 1981. The Kitagawa-Ueda definition is a special case of this definition when direction of mean spin is perpendicular to θ-direction.

The quantum entanglement is one of the most important phenomena in quantum information processing. Quantum entanglement of mixed states has been paid much attention in recent days and widely considered in different physical systems (Wootters, 1998; Peres, 1996; Horodecki, 1997). When we deal with the entanglement of mixed states, first step is to choose a criterion to quantify it. Peres and Horodecki (Peres, 1996; Horodecki, 1997) found a criterion to evaluate the entanglement of mixed state. According to Peres et. al. when the partial transposition of its density matrix gives negative eigenvalues, the bipartite system is entangled. Spin squeezing can also be used as a measure of entanglement in multi-atom system. Sorenson et al., 2002 proposed a criterion to quantify the entanglement in term of spin squeezing parameter. According to the definition of Sorensen et. al., a state having squeezing parameter $\xi < 1$ is an entangled state. In the present paper we study the relation between entanglement and spin squeezing in two two-level systems interacting with a single mode coherent radiation. We conclude that the criterion for spin squeezing given by Prakash and Kumar is better than that given by Sorensen et al. for the measurement of entanglement. It has been found that when the atoms are initially in excited state they become highly entangled through interaction with coherent radiation.

Time Evolution Operator for Two Two-Level Atoms Interacting With a Single Mode Radiation

Consider a system of two two-level atoms interacting with a single resonant mode of radiation with zero detuning. If the atoms are located in a region small in comparison with the wavelength of the field, but not so small so as to make them interact directly with each other, the Hamiltonian (Dicke, 1954) of the system in the dipole and rotating wave approximation is given in the natural system of units ($\hbar = 1$) by

$$H = H_0 + H_I;\ H_0 = H_F + H_A, \tag{6}$$

Here, $H_F = \omega_F N$, $H_A = \omega_A S_z$, $H_I = g(a S_+ + a^+ S_-)$, $N = a^+a$ and subscripts F, A, and I refer to field, atoms and interaction, N, a and a^+ are number, annihilation and creation operators respectively, $S_{\pm z}$ g is coupling constant and are the Dicke's collective atom operators (Dicke, 1954). If $|u\rangle_i$ and $|l\rangle_i$ are the interacting upper and lower energy states of the i[th] $(i = 1, 2)$ two-level atom,

$$S_\pm = \sum_{i=1,2} S_{\pm i};\quad S_z = \sum_{i=1,2} S_{zi};\quad S_\pm \equiv S_{xi} \pm iS_{yi}, \tag{7}$$

$$S_{+i} = |u\rangle_{i\,i}\langle l|;\quad S_{-i} = |l\rangle_{i\,i}\langle u|;$$
$$S_z = \frac{1}{2}[|u\rangle_{i\,i}\langle u| - |l\rangle_{i\,i}\langle l|]; \tag{8}$$

Here $S_\pm$ and S_z satisfy the angular momentum commutation relation,

$$[S_+, S_-] = 2S_z,\ [S_z, S_\pm] = \pm S_\pm. \tag{9}$$

In the truncated Hilbert space, the atomic system is described by the eigenstates $|j,m\rangle$ defined by, $S^2|j,m\rangle = j(j+1)|j,m\rangle$; $S_z|j,m\rangle = m|j,m\rangle$, $S^2 = \frac{1}{2}[S_+S_- + S_-S_+] + S_z^2$. For a system consisting of two two-level atoms, $j = 1$ with $m = 1, 0, -1$ and $j = 0$ with $m = 0$. These two states have the property $S_\pm|j,m\rangle = \sqrt{(j \mp m)(j \pm m + 1)}\,.\,|j, m \pm 1\rangle$ Since $[S^2, H] = 0$, quantum number j does not change and, therefore, if the atomic state has $j = 1$ initially, we have to consider states $|1, 1\rangle$, $|1, 0\rangle$, and $|1, -1\rangle$ only while studying the later time evolution of the atomic state.

We note that commutation relations (9) lead to $[H_0, H_I] = 0$. This shows that the time evolution operator $U = e^{-iHt}$ can be written as, $U = U_0 U_I$ where. $U_0 = e^{-iH_0 t}$ and $U_I = e^{-iH_I t}$

The exact time evolution operator in interaction picture in the form (Prakash and Kumar, 2005),

$$U_I = e^{-iH_I t} = \begin{pmatrix} 1+(N+1)\,C(N+1) & -i\,S(N+1)\,a & C(N+1)\,a^2 \\ -ia^+\,S(N+1) & \cos(gt\sqrt{4N+2}\,) & -i\,S(N)\,a \\ a^{+2}\,C(N+1) & -ia^+\,S(N) & 1+N\,C(N-1) \end{pmatrix} \quad (10)$$

Here

$$C(N) \equiv \{\cos(\,gt\sqrt{4N+2}\,) - 1\}/(2N+1)$$

$$S(N) \;\equiv\; \{\sin\,(gt\,\sqrt{4N+2}\,)\}\;/\sqrt{2N\;+1} \quad (11)$$

Criteria for Spin Squeezing

Instead of considering squeezing of spin components Sx and Sy separately, let us consider a more general operator,

$$S_\theta \;=\; S_x\cos\theta + S_y\sin\theta \quad (12)$$

Commutation relation $[S_\theta\,, S_{\theta+\pi/2}] = i\,S_z$ gives

$$\left\langle(\Delta S_\theta)^2\right\rangle\left\langle(\Delta S_{\theta+\pi/2})^2\right\rangle \ge \frac{1}{4}\left|\langle S_z\rangle\right|^2 \quad (13)$$

Commutation relations $[S_\theta, S_{\theta+\pi/2}] = i\,S_z$ and $[S_\theta, S_z] = -\,i\,S_{\theta+\pi/2}$, indicate squeezing for S_θ, if $\left\langle(\Delta S_\theta)^2\right\rangle < \frac{1}{2}\left|\langle S_Z\rangle\right|$ or $\left\langle(\Delta S_\theta)^2\right\rangle < \frac{1}{2}\left|\langle S_{\theta+\pi/2}\rangle\right|$. We can obtain the most general criterion for squeezing of the operator S_θ by considering, in place of the triad of operators, (S_θ, $S_{\theta+\pi/2}$ and S_z), the triad, (S_θ, $S_{\theta+\pi/2,\,\varphi}$ and $S_{\theta+\pi/2,\,\varphi+\pi/2}$) with

$$S_{\theta+\pi/2,\,\varphi} = S_{\theta+\pi/2}\cos\varphi + S_z\sin\varphi\;;\; S_{\theta+\pi/2,\,\varphi+\pi/2} = -S_{\theta+\pi/2}\sin\varphi + S_z\cos\varphi, \quad (14)$$

and an arbitrary φ. These operators give $[S_\theta, S_{\theta+\pi/2,\,\varphi}] = i\,S_{\theta+\pi/2,\,\varphi+\pi/2}$, $[S_\theta, S_{\theta+\pi/2,\,\varphi+\pi/2}] = -i\,S_{\theta+\pi/2,\,\varphi}$ and therefore the uncertainty relations,

$$\left\langle(\Delta S_\theta)^2\right\rangle\left\langle(\Delta S_{\theta+\pi/2,\varphi})^2\right\rangle \ge \frac{1}{4}\left|\langle S_{\theta+\pi/2,\,\varphi+\pi/2}\rangle\right|^2 \quad (15)$$

$$\left\langle(\Delta S_\theta)^2\right\rangle\left\langle(\Delta S_{\theta+\pi/2,\varphi+\pi/2})^2\right\rangle \ge \frac{1}{4}\left|\langle S_{\theta+\pi/2,\,\varphi}\rangle\right|^2 \quad (16)$$

One may call S_θ squeezed if

$$\left\langle(\Delta S_\theta)^2\right\rangle < \frac{1}{2}\left|\langle S_{\theta+\pi/2,\,\varphi+\pi/2}\rangle\right| \quad \text{and / or}$$

$$\left\langle(\Delta S_\theta)^2\right\rangle < \frac{1}{2}\left|\langle S_{\theta+\pi/2,\varphi}\rangle\right| \quad (17)$$

Eq(17) shows that the values of $\left|\langle S_{\theta+\pi/2,\,\varphi}\rangle\right|$ and $\left|\langle S_{\theta+\pi/2,\,\varphi+\pi/2}\rangle\right|$ as φ is varied, lies between 0 and

$$\left[\langle S_Z\rangle^2 + \langle S_{\theta+\pi/2}\rangle^2\right]^{1/2}$$

The most general criterion for squeezing is, therefore,

$$\left\langle(\Delta S_\theta)^2\right\rangle < \frac{1}{2}\left[\langle S_Z\rangle^2 + \langle S_{\theta+\pi/2}\rangle^2\right]^{1/2} \quad (18)$$

because, if this relation holds, then one can always find separate intervals for φ for holding of the two Eqs.(17). In case these two intervals for φ overlap, in the region of overlap, both of Eqs.(17) are satisfied and both components, S_θ and $S_{\theta+\pi/2}$ are squeezed simultaneously (Prakash and Kumar, 2005) .

Evaluation of Spin Squeezing and Entanglement

Let us consider an interacting system of two two-level atoms and a single mode radiation. We consider distance between atoms small as compared to the radiation wavelength but not so close so as to make them interact with each other directly. The Hamiltonian for this system in the dipole and rotating wave approximations has been has been solved and the time-evolution operator evaluated. We can use the result Eq. (10) for it, find the state of the systems after interaction for time interval t and find spin squeezing.

Before we make explicit calculations for squeezing factors, we note that we can write

$$f(S_\theta) = \langle\alpha\,|\,\langle j,m\,|\,U^{+I} f(S_x) U_I\,|\,j,m\rangle\,|\,\alpha\rangle =$$

$$\langle\alpha\,|\,\langle j,m\,|\,e^{iH_I t} e^{-i\,\theta(S_z+N)} f(S_x)\, e^{i\theta(S_z+N)} e^{-i\,H_I t}\;|\,j,m\rangle\,|\,\alpha\rangle \quad (19)$$

Since $[Sz + N, HI] = 0$ and

$$e^{i\theta N}\;|\,\alpha\rangle = \sum_n e^{-\frac{1}{2}|\alpha|^2}\,\frac{\alpha^n}{\sqrt{n\,!}}\,e^{i\theta N}\;|\,n\rangle$$

$$= \sum_n e^{-\frac{1}{2}|\alpha|^2}\,\frac{(\alpha\,e^{i\,\theta})^n}{\sqrt{n\,!}}\,|\,n\rangle = |\,\alpha\,e^{i\,\theta}\rangle \quad (20)$$

$$f(S_\theta)= \langle \alpha | \langle j, m | e^{-i\theta (S_z+N)} e^{iH_I t} f(S_x) e^{-iH_I t} e^{i\theta (S_z+N)} | j, m\rangle | \alpha\rangle$$

$$= \langle |\alpha| e^{-i(\theta+\theta_\alpha)} | \langle j, m | e^{iH_I t} f(S_x) e^{-iH_I t} | j, m\rangle | |\alpha| e^{i(\theta+\theta_\alpha)} \rangle \quad (21)$$

This shows that, when initial radiation state is $|\alpha\rangle = ||\alpha|e^{i\theta_\alpha}\rangle$ squeezing factor for S_θ is a function of $\theta + \theta_\alpha$ and hence the results for arbitrary θ and θ_α can be obtained by studying squeezing of S_θ with (i) θ arbitrary but θ_α fixed (i.e., θ_α and θ real) or (ii) θ_α arbitrary and θ fixed (i.e., $\theta = 0$ and $S_\theta=S_x$).

(a) Spin Squeezing

If both atoms are excited and radiation is in the coherent state $|\alpha\rangle$ initially, the initial state is, and the final state is then obtained using Eq. (10) in the form,

$$|\psi\rangle = \left[1+(N+1)C(N+1)\right]|\alpha\rangle|1,1\rangle - ia^{+} S(N+1)|\alpha\rangle|1, 0\rangle + a^{+2} C(N+1)|\alpha\rangle|1,-1\rangle \quad (22)$$

Direct results using Eq.(21) and Eq.(22) are,

$$\langle S_\theta\rangle = \sqrt{2}\, |\alpha| \sin(\theta+\theta_\alpha)(P_1 - P_2)$$

$$\langle S_{\theta+\pi/2}\rangle = \sqrt{2}\, |\alpha| \cos(\theta+\theta_\alpha)(P_1 - P_2) \quad (23)$$

$$\langle S_z\rangle = Q_1 - Q_2$$

$$\langle S_\theta^2\rangle = \frac{1}{2} + \frac{1}{2}R_1 + |\alpha|^2 \cos 2(\theta+\theta_\alpha) R_2 \quad (24)$$

Where,

$$P_1 = e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!}(n+2)S(n+2)C(n+1),$$

$$P_2 = e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!}(1+(n+2)C(n+2))\, S(n+1),$$

$$Q_1 = e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!}(1+(n+1)C(n+1))^2,$$

$$Q_2 = e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!}(n+1)(n+2)(C(n+1))^2,$$

$$R_1 = e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!}(n+1)(S(n+1))^2,$$

$$R_2 = e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!}(1+(n+3)C(n+3))C(n+1),$$

and C(n) and S(n) are as defined by Eq.(11), if we replace N by n. Using Eq.(18), we define the squeezing factor,

$$S_\theta = \left\langle (\Delta S_\theta)^2 \right\rangle / \tfrac{1}{2}[\langle S_z\rangle^2 + \langle S_{\theta+\pi/2}\rangle^2]^{1/2} \quad (25)$$

which is a function of $|\theta + \theta_\alpha|$, α and gt. This gives spin squeezing for a general spin component S_θ, whenever $S_\theta < 1$.

When we varied even $\theta + \theta_\alpha$ (i.e. when we dropped the Kitagawa-Ueda restrictions that squeezing of only spin components perpendicular to mean atomic spin will be considered) and considered all spin components, we find the minimum squeezing factor $S_\theta = 0.09612$ (i.e. nearly 90% squeezing) at $\theta = -\theta_\alpha + \pi/2$, gt = 0.029 and $\alpha = 26.247$. We study variation of S_θ with gt for fixed $\theta + \theta_\alpha$ and α near this minimum and results are given in Figure 1. We also compare the results of spin squeezing with that of Sorensen et al. results in Figure 1.

(b) Entanglement

The Dicke system is composed of three energy levels $|1,1\rangle = |e_1 e_2\rangle$, $|1,0\rangle = (|e_1 g_2\rangle + |e_2 g_1\rangle)/\sqrt{2}$ and $|1,-1\rangle = |g_1 g_2\rangle$. These energy levels are known as the Dicke collective states. The states $|1,1\rangle$ and $|1,-1\rangle$ are the excited $|e\rangle$ and ground state $|g\rangle$ levels of individual atoms whereas $|1,0\rangle$ is a superposition of $|e\rangle$ and $|g\rangle$ state of individual atom. In order to analyze the relation between entanglement and spin squeezing parameter, we express the density matrix, $\rho = |\psi\rangle\langle\psi|$ of the system. Using Eq.(22), we find the density matrix of the system in basis of the product states $\{|e_1 e_2\rangle, |e_1 g_2\rangle, |e_2 g_1\rangle, |g_1 g_2\rangle\}$ which takes the form,

$$\rho = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{21} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} \quad (26)$$

Here,

$$m_{11} = e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!}[1+(n+1)C(n+1)]^2,$$

$$m_{12} = -i\alpha^* e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!}[1+(n+2)C(n+2)]S(n+1),$$

$m_{13} = m_{12}$,

$$m_{14} = \alpha^2 e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!}[1+(n+3)C(n+3)]C(n+1),$$

$m_{21} = (m_{12})^*$,

$$m_{22} = \tfrac{1}{2} e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!}(n+1)[S(n+1)]^2 = m_{23} = m_{32} = m_{33},$$

$$m_{24} = \frac{\alpha^*}{\sqrt{2}} e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!}[(n+2)C(n+1)]S(n+2) = m_{23},$$

$m_{31} = (m_{13})^*$, $m_{41} = (m_{14})^*$, $m_{42} = (m_{24})^*$, $m_{43} = (m_{34})^*$

and

$$m_{44} = e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!}(n+1)(n+2)[C(n+1)]^2$$

Given the density matrix, it is possible to calculate the entanglement between the atoms. To quantify the degree entanglement, we use the negativity criterion for entanglement (Wootters, 1998), where negativity is the sum of the absolute value of all the negative eigen values of partial transposition density matrix. The partial transposition of density matrix, $\rho = |\psi\rangle\langle\psi|$ is

$$\rho^T = \begin{pmatrix} m_{11} & m_{12} & m_{31} & m_{32} \\ m_{21} & m_{21} & m_{41} & m_{42} \\ m_{13} & m_{14} & m_{33} & m_{34} \\ m_{23} & m_{24} & m_{43} & m_{44} \end{pmatrix}$$

We calculate numerically the eigen values of partial transposition of density matrix and find the entanglement

$$E = \log_2(2N+1) \qquad (27)$$

where N the is negativity.

DISCUSSION

The variation of entanglement with coupling constant gt is shown in Figure (1). Figure (1) clearly shows that the spin squeezing defined on the basis of Prakash criterion (HPC) measures the entanglement for small coupling time and vice versa. From Figure (1), we conclude that for small coupling time both entanglement and spin squeezing is maximum simultaneously while for large coupling time both do not occur simultaneously. The Sorenson criterion (SOCI) does not explain this effect as shown in Figure (1). Figure (2) shows variation of entanglement with coupling time gt and square root of average number of photons for $\theta + \theta_\alpha = \pi/2$.

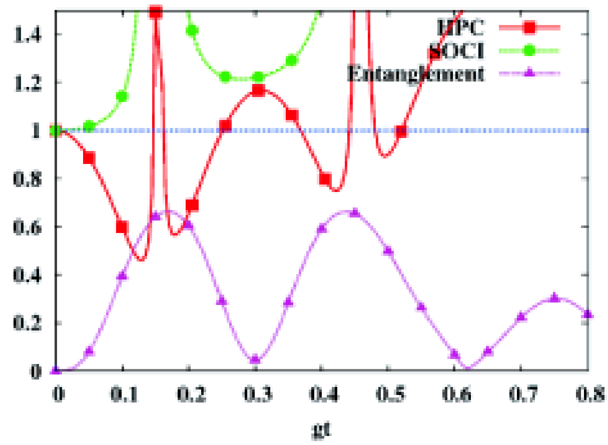

Figure 1 : Variation of Squeezing factor, HP Criterion (HPC), Sorension Criterion (SOCI) and Entanglement with coupling time gt for fixed $|\alpha|$ and $\theta + \theta_\alpha = 0$ for the state $|1, 1\rangle$

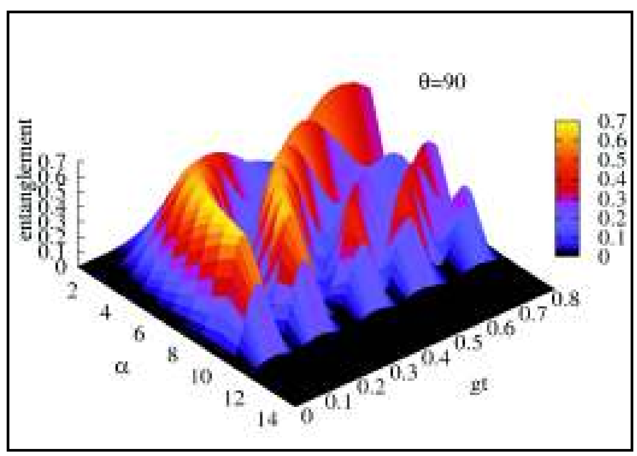

Figure 2 : Variation of Entanglement with $|\alpha|$, coupling time gt for fixed $\theta + \theta_\alpha = \pi/2$.

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