Indian J.Sci.Res.3(2) : 183-186, 2012

CERTAIN INTEGRALS INVOLVING G-FUNCTION OF TWO VARIABLES

D. KANDU[1]

Department of Applied Sciences,M.M.M. Engineering College,Gorakhpur,U.P., India
E-mail: kandudevendra891@gmail.com

ABSTRACT

In this paper, our aim is to evaluate certain double integrals involving G-function of two variables.

KEYWORDS : Double integrals, G-function, Hypergeometric function

Singh (1977) and Denis (1970), evaluated certain single integrals involving generalized hypergeometric functions of two variables.

He has used the following summation formula deducted by Carlitz, L.

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(a)_m(a')_n(b)_{m+n}(c)_m(c')_n}{(1)_m(1)_n(b)_m(b')_n(c+c')_{m+n}} = \phi\Gamma\begin{bmatrix} c-a', c'-a, b-a-a', c+c' \\ c+c'-a-a', b-a, b-a', c, c' \end{bmatrix} \quad \ldots(1)$$

Evaluating double integrals involving G-function of two variable, the above summation formula (1) will be used.(Denis,1972)

The main results to be established are :

$$\int_0^1\int_0^1 \lambda^{c-1}(1-\lambda)^{c'-1}\mu^{d-1}(1-\mu)^{d'-1}\,.F_2\begin{bmatrix} b: a, a'; \\ -: b, b; \end{bmatrix}\lambda, 1-\lambda\Bigg]$$

$$F_2\begin{bmatrix} e: f, f'; \\ -: e, e; \end{bmatrix}\mu, 1-\mu\Bigg].G^{\,n,v_1,v_2,m_1,m_2}_{\,p,[t,t'],s,[q,q']}\left[\begin{matrix} x\lambda\mu \\ y(1-\lambda)(1-\mu)\end{matrix}\left|\begin{matrix}\epsilon_p \\ (\Upsilon_t);(\Upsilon'_{t'}) \\ (\delta_s) \\ (\beta_q);(\beta'_{q'})\end{matrix}\right.\right]d\lambda\, d\mu$$

$$=\Gamma\begin{bmatrix} b, e, b-a-a', e-f-f' \\ b-a, e-f, b-a', e-f' \end{bmatrix}\times G^{n,v_1+2,v_2+2,m_1,m_2}_{p,[t+z,t'+z,s+z,[q,q']}$$

$$\left[\begin{matrix} x \\ \\ y\end{matrix}\left|\begin{matrix} & (\epsilon_p) \\ c-a', d-f', & (\Upsilon_t); c'-a, d'-f\ \ ,(\Upsilon'_{t'}) \\ & (\delta_s), c+c'-a-a', d+d'-f-f' \\ & (\beta_q);(\beta'_{q'})\end{matrix}\right.\right] \quad \ldots(2)$$

provided that

$$\begin{bmatrix} 0\le m_1\le q, 0\le m_2\le q', 0\le v_1\le t, 0\le v_2\le t, 0\le n\le p \\ p+q+s+t<2(m_1+v_1+n), p+q'+s+t'<2(m_2+v_2+n) \\ and \\ |arg x|<\pi\left[m_1+v_1+n-\frac{1}{2}(p+q+s+t)\right], \\ |arg y|<\pi\left[m_2+v_2+n-\frac{1}{2}(p+q'+s+t')\right], \\ Rl(c'c', d, d')>0 \end{bmatrix} \quad \ldots(3)$$

[1]Corresponding author

$$\int_0^1\int_0^1 \lambda^{c-1}(1-\lambda)^{c'-1}(1-\mu)^{d'-1}\mu^{d-1}\cdot F_2\left[{b:a,a'; \atop -:b,b;}\lambda,1-\lambda\right].$$

$$.F_2\left[{e:f,f'; \atop -:e,e;}\mu,1-\mu\right].G^{\,n,v_1,v_2,m_1,m_2}_{\,p,[t,t'],s,[q,q']}\left[\begin{matrix} x\lambda\mu \\ \\ y(1-\lambda)(1-\mu)\end{matrix}\left|\begin{matrix}\epsilon_p \\ (\Upsilon_t);(\Upsilon'_{t'}) \\ (\delta_s) \\ (\beta_q);(\beta'_{q'})\end{matrix}\right.\right]d\lambda\, d\mu$$

$$=\Gamma\left[{b,e,b-a-a',e-f-f' \atop b-a',e-f',b-a\ \ ,e-f}\right]\times G^{n,v_1+2,v_2+2,m_1,m_2}_{p,[t+z,t'+z,s+z,[q,q']}$$

$$\left[\begin{matrix} x \\ \\ y\end{matrix}\left|\begin{matrix} & (\epsilon_p) \\ c-a',d-f\ \ , & (\Upsilon_t);c'-a,d'-f\ \ ,(\Upsilon'_{t'}) \\ & (\delta_s),c+c'-a-a',d+d'-f-f' \\ & (\beta_q);(\beta'_{q'})\end{matrix}\right.\right] \qquad ...(4)$$

Provided that conditions (3) hold (Denis,R.Y,1972)

(i) $\int_0^1\int_0^1 \lambda^{c-1}(1-\lambda)^{c'-1}\ \mu^{d-1}\cdot(1-\mu)^{d'-1}.F_2\left[{b;a,a'; \atop -;b,b;}\lambda,1-\lambda'\right]..F_2\left[{e:f,f'; \atop -:e,e;}\mu,1-\mu\right]$

$$.G^{\,n,v_1,v_2,m_1,m_2}_{\,p,[t,t'],s,[q,q']}\left[\begin{matrix} x\lambda\mu \\ \\ y\lambda\mu\end{matrix}\left|\begin{matrix}\epsilon_p \\ (\Upsilon_t);(\Upsilon'_{t'}) \\ (\delta_s) \\ (\beta_q);(\beta'_{q'})\end{matrix}\right.\right]d\lambda\, d\mu$$

$$=\Gamma\left[{a'-a,d'-f,b,b-a-a',e,e-f-f' \atop b-a,b-a',e-f,e-f'}\right]\times G^{n+2,v_1,v_2,m_1,m_2}_{p+2,[t,t'],s+z,[q,q']}$$

$$\left[\begin{matrix} x \\ \\ y\end{matrix}\left|\begin{matrix} 1-c+a',\ a-d+f',(\epsilon_p) \\ (\Upsilon_t)\qquad\qquad;(\Upsilon'_{t'}) \\ (\delta_s),c+c'-a-a',d+d'-f-f' \\ (\beta_q);(\beta'_{q'})\end{matrix}\right.\right] \qquad ...(5)$$

provided $0\le m_1\le q, 0\le v_1\le t, t+q\le z(m_1+v_1)|\arg x|<\pi[m_1+v_1-\frac{1}{2}(t+q)]$

and $Rl(c,c',d,d')>0$.

(ii) If we put $p=s=0,t=t',q=q',m_2=1\beta'_1=0=y,t\le q$ in (4) , we get

$$\int_0^1\int_0^1 \lambda^{c-1}(1-\lambda)^{c'-1}\ \mu^{d-1}\cdot(1-\mu)^{d'-1}.F_2\left[{b;a,a'; \atop -;b,b;}\lambda,1-\lambda\right]\times\left[{e:f,f'; \atop -:e,e;}1-\mu,\mu\right]$$

$$.G^{\,v_1,,m_1}_{\,t,q}\left[x\lambda\mu\left|\begin{matrix}1-(\Upsilon)_t \\ (\beta_q)\end{matrix}\right.\right]d\lambda\, d\mu$$

$$=\Gamma\begin{bmatrix} b,e,b-a-a',e-f-f' \\ b-a,,e-f',b-a,e-f \end{bmatrix} \times G^{\nu_1+2,m_1}_{t+2,q+2}\left[x\middle| \begin{matrix} 1-(\Upsilon_t),1+a'-c,1+f-d \\ (\beta_q),1-c-c'+a+a',a-d-d'+f+f' \end{matrix}\right]$$

Provided that $0 \le m_1 \le q, 0 \le \nu_1 \le t, t+q \le z(m_1+\nu_1)|\arg x| < \pi[m_1+\nu_1-\frac{1}{2}(t+q)]$.

(iii) let us take $p=s=0,t=t',q=q',m_2=1,\beta'_1=0,y=0,t \le q$ in result (5) we get:

$$\int_0^1\int_0^1 \lambda^{c-1}(1-\lambda)^{c\prime-1}\ \mu^{d-1}\cdot(1-\mu)^{d\prime-1}.F_2\begin{bmatrix} b;a,a'; \\ -;b,b; \end{bmatrix}\lambda,1-\lambda\Big]..F_2\begin{bmatrix} e:f,f'; \\ -:e,e; \end{bmatrix}\mu,1-\mu\Big]$$

$$.G^{\nu_{1,},m_1}_{t,q}\left[x\lambda\mu\middle| \begin{matrix} 1-(\Upsilon)_t \\ (\beta_q) \end{matrix}\right] d\lambda\, d\mu$$

$$=\Gamma\begin{bmatrix} c'-a,d'-f,b,b-a-a',e,e-f-f' \\ b-a,b-a',e-f\ \ ,e-f' \end{bmatrix} \times G^{\nu_1+2,m_1}_{t+2,q+2}$$

$$\left[x\middle| \begin{matrix} 1-c+a',1-d+f',1-(\Upsilon_t) \\ (\beta_q),1+a+a'-c-c',1+f+f'-d-d' \end{matrix}\right]$$

Provided that above conditions hold.(Agrawal, 1965)

(iv) If we take $p=s=0,t=t',m_2=1,\beta' \ \ =0=y,t \le q$ in result (6) , we get :

$$\int_0^1\int_0^1 \lambda^{c-1}(1-\lambda)^{c\prime-1}\ \mu^{d-1}\cdot(1-\mu)^{d\prime-1}.F_2\begin{bmatrix} b;a,a'; \\ -;b,b; \end{bmatrix}\lambda,1-\lambda\Big]..F_2\begin{bmatrix} e:f,f'; \\ -:e,e; \end{bmatrix}\mu,1-\mu\Big]$$

$$.G^{\nu_1,m_1}_{t,q}\left[x(1-\lambda)(1-\mu)\middle| \begin{matrix} 1-(\Upsilon)_t \\ (\beta_q) \end{matrix}\right] d\lambda\, d\mu$$

$$\int_0^1\int_0^1 \lambda^{c-1}(1-\lambda)^{c\prime-1}\ \mu^{d-1}\cdot(1-\mu)^{d\prime-1}.F_2\begin{bmatrix} b;a,a'; \\ -;b,b; \end{bmatrix}\lambda,1-\lambda\Big].\times F_2\begin{bmatrix} e:f,f'; \\ -:e,e'; \end{bmatrix}\mu,1-\mu\Big]$$

$$.G^{n,\nu_1,\nu_2,m_1,m_2}_{p,[t,t'],s,[q,q']}\left[\begin{matrix} x(1-\lambda)(1-\mu) \\ \\ y(1-\lambda)(1-\mu) \end{matrix}\middle| \begin{matrix} \epsilon_p \\ (\Upsilon_t);(\Upsilon'_{t'}) \\ (\delta_s) \\ (\beta_q);(\beta'_{q'}) \end{matrix}\right] d\lambda\, d\mu$$

$$=\Gamma\begin{bmatrix} c-a',d\ \ -f',b,b-a-a',e,e-f-f' \\ b-a,b-a',e-f,e-f' \end{bmatrix} \times G^{n+2,\nu_1,\nu_2,m_1,m_2}_{p+2,[t,t'],s+2,[q,q']}$$

$$\left[\begin{matrix} x \\ \\ y \end{matrix}\middle| \begin{matrix} 1-c+a',\ a-d+f',(\epsilon_p) \\ (\Upsilon_t)\qquad ;(\Upsilon'_{t'}) \\ (\delta_s),c+c'-a-a',d+d'-f-f' \\ (\beta_q);(\beta'_{q'}) \end{matrix}\right] \qquad ...(6)$$

Provided that conditions (3) hold.

PROOF OF RESULT (2) to (6)

To prove these results, we replace the g-function of two variable on the left hand side of the results by their equivalent double integrals and integrating them term by term which are admissible due to the absolute convergence of the series and the integrals involved; under the given conditions and finally summing their quadriple series with the help of results (1) ,we get the right hand side of the results.

PARTICULAR CASES :

(i) Putting, $p = s = 0, t = t', q = q', m_2 = 1, \beta' = 0, y = 0, t \leq q$ in result (2) , we get :

$$\int_0^1 \int_0^1 \lambda^{c-1}(1-\lambda)^{c'-1} \ \mu^{d-1} \cdot (1-\mu)^{d'-1} . F_2\left[\begin{matrix} b; a, a'; \\ -; b, b; \end{matrix} \lambda, 1-\lambda\right] . F_2\left[\begin{matrix} e: f, f'; \\ -: e, e; \end{matrix} \mu, 1-\mu\right]$$

$$. G_{t,q}^{m_1 v_1}\left[x\lambda\mu \left| \begin{matrix} 1-(\Upsilon)_t \\ (\beta_q) \end{matrix} \right.\right] d\lambda \, d\mu$$

$$= \Gamma\left[\begin{matrix} , b, e, b-a-a', e-f-f' \\ b-a', b-a, e-f \ \ , e-f' \end{matrix}\right] \times G_{t+2,q+2}^{m_1, v_1+2}$$

$$\left[x \left| \begin{matrix} 1-(\Upsilon_t) \ ; 1-c+a', 1-d+f' \\ (\beta_q), 1+a+a'-c-c', 1+f+f'-d-d' \end{matrix} \right.\right]$$

$$= \Gamma\left[\begin{matrix} c-a', d-f', b-a-a', e-f-f' \\ b-a, b-a', e-f, e-f' \end{matrix}\right] \times G_{t+2,q+2}^{v_1+2, m_1}$$

$$\left[x \left| \begin{matrix} 1-c'+a \ \ , 1-d'+f \ \ , 1-(\Upsilon_t) \\ (\beta_q), 1+a+a'-c-c', 1+f+f'-d-d' \end{matrix} \right.\right]$$

provided that the above conditions hold.

REFERENCES

Agrawal R.P.; 1965. An Extension of Meijer's g-functions, Proc. Nat. Inst. Sc .(India),A,**6,31**:536-46**.**

Denis R.Y.; 1970. Certain integrals involving G-function of two variables.Jour.Ganita,**21(2):**1-10**.**

Denis R.Y.; 1972.; An integrals involving generalized hypergeometric functions . Math Student,A,**30**;79-81.

Singh V.N.:1977. Certain integrals involving generalized hypergeometric functions of two variables.Jour. Acta Cia. Indica, **3**:189-191