FREE VIBRATION ANALYSIS OF LAMINATED COMPOSITE BEAMS

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ABSTRACT

These Laminated composite sections possess high stiffness to weight ratio with enhanced durability characteristics as compared to their monolithic counterparts. Accordingly, laminated composite structures have found increased utility as structural elements during the recent years. But these light weight structures are likely to undergo large amplitude vibration leading to failure of the structure. This necessitates to predict the vibration characteristics of these structures accurately. This paper presents the free vibration analysis of laminated composite beams. Finite element method of analysis is adopted using ANSYS 15.0 software. Effect of different parameters on the dynamic characteristics of the laminated composite beam influencing its natural frequency has been investigated and results are presented.

KEYWORDS: Laminated composite beam, Finite Element Analysis, ANSYS, Free Vibration.

Over the last few decades, considerable increase in the use of laminated composite beams has been found in different fields of engineering due to their excellent features. Laminated composite beams are notable for their high in-plane stiffness, bending stiffness, strength, coefficient of thermal expansion, fatigue resistance along with high strength or stiffness to weight ratio. Above all, the properties of these laminated composite beams can be tailor made to meet specific design requirements. However, being light weight, they are prone to undergo large deformation and excessive vibration. Hence, it is very much important to predict the vibration behaviour of these laminated composite beams accurately. A lot of studies have been carried out in the past to predict the behaviour of these laminated composite beams. Based on Timoshenko beam theory, Jafari-Talookolaei et al., 2012 carried out free vibration analysis of laminated composite beams using Lagrange multipliers. Mohammad-Abadi and Daneshmehr, 2015 carried out comparison of Euler- Bernoulli, Timoshenko and Reddy beam theory for the free vibration analysis of laminated composite beams. Qu et al., 2015 presented exact three dimensional elasticity theory to determine the free and transient vibration response of laminated composite structures with parallelepiped shapes like beams, plates and solids. Mareishi et al., 2014 studied free vibration, large static deflection, mechanical and thermal buckling and post buckling of laminated composite beams with surface bonded piezoelectric fiber reinforced composite layer under a combined mechanical, thermal and electrical loading. Jafari-Talookolaei et al., 2016 have presented analytical and finite element solutions for free vibration analysis of delaminated composite curved beams considering the effect of shear deformation, rotary inertia, deepness and material coupling. Based on one dimensional beam finite element analysis, Sheikh and Asadi, 2015 proposed a technique for vibration of thin walled laminated composite beam having closed section. Filippi et al., 2015 presented a refined beam theory for static and dynamic analysis of laminated composite beams using higher order expansion of Cheryshev polynomials and Carrera unified formulation for finite element analysis. Pagani et al., 2014 used Carrera unified formulation to obtain higher order beam theories for laminated composite beams and obtained the equation of motion and natural boundary conditions in free vibration. Li et al., 2014 made comparison of various shear deformation beam theory for free vibration of laminated composite beams. Boay and Wee, 2008 developed a closed form expression to determine an effective flexural modulus for the free vibration response, bending and buckling analysis and validated the results through ANSYS. Li and Qiab, 2014 presented a geometrically nonlinear free vibration analysis of shear deformable anisotropic laminated composite beams resting on elastic foundation. Vo and Thai, 2012 presented the free vibration analysis of axially loaded laminated rectangular composite beams using refined shear deformation theory. Again, Vo et al., 2011 developed a general analytical model for the free vibration analysis of thin-walled laminated composite I-beam. Ghoneam, 1995 discussed the dynamic characteristics of laminated composite beam with various fiber orientation and different boundary conditions in the absence and presence of crack. Arya, 2003 presented a new type of zig-zag displacement function used for free vibration analysis of a thick simply supported laminated composite beam. Su and Della, 2004 presented an analytical solution to the free vibration of composite beams with two non-overlapping delamination with experiment and analysis. Sisi et al., 2015 presented a theoretical method for low velocity impact analysis of laminated composite beams with arbitrary lay-ups and various boundary conditions subjected to asynchronous

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or repeated impacts of multiple masses. Vo and Lee, 2009 studied a general analytical model based on classical lamination theory applicable to flexural-torsional coupled vibration of thin walled composite box beams. Vo and Lee, 2008 studied free vibration of a thin-walled laminated composite beam with a general analytical model based on classical lamination theory, applicable to the dynamic behavior of the thin-walled composite box section. Jun et al., 2008 presented a dynamic finite element method for free vibration analysis of generally laminated composite beams based on first order shear deformation theory. Jun and Hongxing, 2009 presented the dynamic stiffness matrix applied to obtain the natural frequencies and mode shapes of laminated composite beam by using Wittrick-Williams algorithm. Song et al., 2015 proposed a spectral element model two- layer smart composite Timoshenko beams consist of a host composite beam and a piezoelectric transducer layer with ANSYS. He and Yang, 2015 applied Kant's higher order beam kinematics, includes both longitudinal and transverse higher order deformation beam to the dynamic model of two-layer partial interaction composite beam and the comparison was done by results of ABAQUS. Emam and Nayfeh, 2009 presented an exact postbuckling configurations of composite beam and the static response represented by the postbuckling and the dynamic response was represented by free vibration analysis. Latifi et al., 2016 studied the nonlinear dynamic responses of symmetric laminated composite beams subjected to combine inplane and lateral loading using full layerwise theory based on advanced first order shear deformation theory. Taufik et al., 1999 presented a theoretical modelling for calculating stiffness and stresses of a composite beam with arbitrary cross section. Luu et al., 2015 developed the NURBS-based isogeometric analysis for the free vibration analysis of the generally laminated Timoshenko- type of deep curved beams with arbitrary curvature. Lee and Kim, 2002 developed a general analytical model based on classical lamination theory taking in coupling of flexural and torsonal modes applied to the dynamic behavior of a thin-walled I-section laminated composite beam. Subramanian, 2006 has used two higher order displacement based on shear deformation and finite element theories for analysis of free vibration of laminated composite beam. Singh et al., 2006 developed a low order analytical model for analysis of non-uniform composite beams which solves the dynamic behavior of the continuous system accurately and the associated direct and inverse problems. Jun et al., 2008 presented the free vibration and buckling behaviors of axially loaded laminated composite beams with arbitrary lay-up using dynamic stiffness method. Calim, 2009 studied analysis of free and forced vibrations of non-uniform composite beams in the Laplace domain and adopted the Timoshenko beam theory. Ghayour et al., 2010 presented a hierarchical finite element model for the flapwise bending vibration analysis of a tapered rotating multilayer composite beam using shear and rotary inertia effects based on higher shear deformation theory. Tekili et al., 2015 analysed the free vibration of beams with composite coats by finite element method. Keeping in view of the importance of the dynamic analysis of the laminated composite beams, in the present paper, it is intended to undertake the parametric analysis pertaining to the free vibration response of a laminated composite beam by finite element method using ANSYS APDL code. The effects of various parametric variations on the free vibration response of the laminated composite beam have been investigated and corresponding results are presented.

FINITE ELEMENT MODELLING

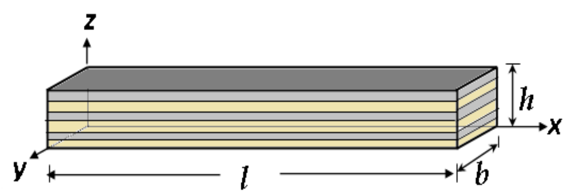

Figure 1: Schematic layout of a laminated composite beam

Fig. 1 represents the schematic diagram of a laminated composite beam consisting of N number of layers having different fiber orientation sequence. The length, width and overall thickness of the beam are **l**, **b** and **h** respectively. The differential equation for the free vibration response of the beam is given by $\left[K\right]-\omega_n^2\left[M\right]=0$. The Finite Element Model has been developed using ANSYS 15.0 software using 4 node 181 shell element. Shell 181element which is a 4 noded element with 6 degree of freedom at each node i.e. translation in X, Y, Z direction and rotation about X, Y, Z axis is ideally suited for analyzing thin to moderately thick structures and possesses the ability for modeling composite and sandwich structures with layered applications A typical model of the laminated composite beam developed using ANSYS 15.0 is depicted in Fig.2

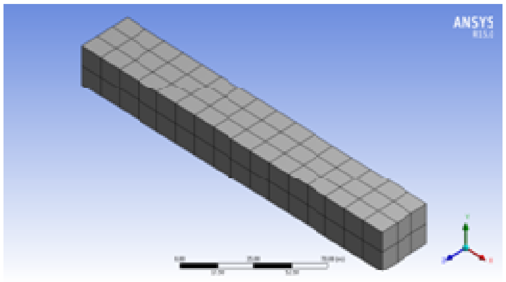

Figure 2: ANSYS model of the Laminated composite beam with meshing

RESULTS AND DISCUSSION

To validate the solution technique used in the present FE model developed using ANSYS APDL, the obtained results using this approach are compared with the results available in literature [Lee et. al., 2014]. For this purpose, the material properties and layup sequence for the beam are considered same as in the reference literature. The comparison of the natural frequency as obtained by the present model and the reference are shown in Table -I.

Table I: Comparison of ANSYS results with reference

Boundary Conditions	Mode-I		Mode-II	
	Present	Ref. [9]	Present	Ref. [9]
(F –F)	659.74	659.3	1749.9	1738.6
(C –F)	106.40	105.3	645.74	637.6
(C –C)	659.31	638.5	1705.0	1657.3
(C – S)	459.30	450.5	1416.9	1389.9
(S –S)	300.25	249.8	1133.9	1132.4

F- Free , C – Clamped, S-Simple Supported

It is observed that the results obtained by the present model are in very good agreement with those in the reference. The accuracy of the developed model being validated, next the parametric evaluation of the free vibration response of the laminated composite beam is carried out. Different parametric variation like fiber orientation angles, boundary conditions, lamination sequences, ratio of elastic modulii etc. have been considered for the free vibration study of the laminated composite beam and the results are presented in the subsequent tables. For all cases the natural frequencies are presented in dimensionless form as $\Omega = \omega L^2(\rho/E_1 h^2)^{1/2}$.

The following material properties are considered for the present study.

E_1 =172.5GPa,, $E_2 = E_3$=0.04 E_1, $G_{12} = G_{13}$= 0.5 E_2, G_{23} = 0.2 E_2, $\gamma_{12} = \gamma_{23} = \gamma_{13} = 0.25$, $\rho = 1600$ kg/m^3.

A. Effect of L/h on the natural frequency of the laminated composite beam

This section presents the variation of natural frequency of a $0^0/90^0/90^0/0^0$ laminated composite beam for various L/h values for different boundary conditions. The results have been shown in Fig. 3.

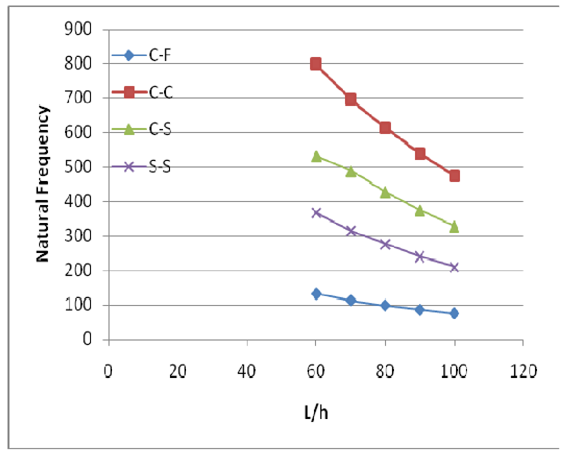

Figure 3: Variation of natural frequency of a $0^0/90^0/90^0/0^0$ laminated composite beam for various L/h values for different boundary conditions.

B. Effect of lamina sequence on the natural frequency of the laminated composite beam

This section presents the variation of natural frequency of a laminated composite beam for various lamina sequences for different boundary conditions corresponding to L/h=100. The results have been shown in Table-II

Table II: Variation of natural frequency for different lamina sequence and boundary conditions (L/h=100)

Boundary conditions	0°/90°/90°/0°	0°/90°/0°/90°	45°/-45°/-45°/45	45°/-45°/45°/-45°	45°/45°/45°/45°
C-F	74.876	52.814	22.014	22.583	18.973
C-C	472.59	336.22	147.72	151.75	125.98
C-S	327.60	232.08	97.591	99.982	85.480
S-S	210.60	148.70	60.686	61.203	58.147
S-F	329.00	232.37	93.337	95.718	81.003

C. Effect of L/b on the natural frequency of the laminated composite beam

This section presents the variation of natural frequency of a $0^0/ 90^0/ 90^0/ 0^0$ laminated composite beam for various L/b values for different boundary conditions corresponding to L/h=100. The results have been shown in Fig.4.

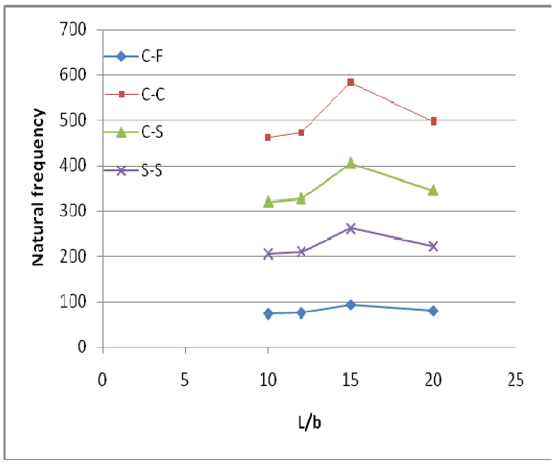

Figure 4: Variation of natural frequency of a $0^0/ 90^0/ 90^0/ 0^0$ laminated composite beam for various L/b values for different boundary conditions corresponding to L/h=100.

D. Effect of L/b on the natural frequency of the laminated composite beam

This section presents the variation of natural frequency of a $0^0/ 90^0/ 90^0/ 0^0$ laminated composite beam for various E_1/E_2 values for different boundary conditions corresponding to L/h=100. The results have been shown in Table-III

Table III: Variation of natural frequency $0^0/ 90^0/ 90^0/ 0^0$ laminated composite beam for various E_1/E_2 values for different boundary conditions corresponding to L/h=100

Boundary conditions	E_1/E_2				
	25	30	35	40	45
C-F	73.136	73.093	73.062	73.038	73.019
C-C	461.61	461.35	461.17	461.02	460.91
C-S	319.98	319.80	319.67	319.56	319.48
S-S	205.70	205.58	205.50	205.43	205.38
S-F	321.35	321.16	321.03	320.93	320.85

CONCLUSION

The parametric studies pertaining to the free vibration analysis of the laminated composite beam have been carried out. Comparison of natural frequency of the laminated composite beams based on different boundary conditions and different lamina lay-up sequences with variation in the L/h ratio, L/b ratio and E_1/E_2 ratio using finite element method in the ANSYS 15.0 software package platform have been carried out and the results are presented which will serve as a guideline for further research in the field of vibration characteristics of laminated composite beams.

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