

N AND Δ BARYON RESONANCES IN CONSTITUENT QUARK MODEL

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ABSTRACT

The mass spectra of *N* and Δ resonances is studied within the framework of a constituent quark model. the confinement potential is assumed as a combination of coulomb like term, a linear confining term plus a harmonic oscillator potential. the results of mass spectra of *N* and Δ resonances is found to be in excellent agreement with the experimental values reported by particle data group over other theoretical model predictions.

KEYWORDS : Constituent Quark Models, Phenomenological Quark Model, Potential Model, Quantum Chromo Dynamics (QCD)

The description of baryons within the constituent quark model is a very important problem in the quantum chromodynamics (QCD). Since the baryon is a three body system its theory is much more complicated than the two-body problem. the baryon spectrum is usually described well by different constituent quark models (Giannini, 1991; Bijker et al., 1994 and Glozman and Riska, 1996). These models have in common fact that the three quark interaction can be separated by two parts: The first one containing the confinement interaction is spin and flavour independent and it is therefore SU(Giannini et al., 2005) invariant ,while the second violates the SU(Giannini et al., 2005) symmetry.

In this paper we study the symmetries of baryon spectrum using a very simple approach based on Gursev - Redicati mass formula (Giannini et al., 2005) The model propose is a simple constituent quark models, where the SU(Giannini et al., 2005) invariant part of the Hamiltonian is the same as hypercentral constituent quark model (Ferrars et al., 1995) and where the SU (Giannini et al., 2005) symmetry is broken by Gursev-Redicati inspired interaction.

METHODOLOGY

In the present work for the study of the non strange baryon mass spectra, we have considered the following Hamiltonian.

$$H = \frac{P_p^2}{2m} + \frac{P_\lambda^2}{2m} + V(r) \tag{1}$$

for the complete description of non strange baryons, we have considered a interaction potential of the form

$$V(r) = V_{H.O.}(r) + V_{CONF.}(r) - V_{HYC.}(r) + V_o \tag{2}$$

Where $V_{H.O.}(r)$ is a added six dimensions harmonic oscillator potential, which has a two body character, and turns out to be exactly hypercentral

$$V_{H.O.}(r) = \sum_{i<j}^3 \frac{1}{2} k(r_i - r_j)^2 = \frac{3}{2} kx^2 = \eta x^2 \tag{3}$$

$V_{CONF.}(r)$ is hyper- linear term gives rise quark confinement (Glozman and Riska,1996) due to large separations.

$$V_{CONF.}(r) = kr \tag{4}$$

$V_{HYC.}(r)$ is a six dimensional hyper coulomb potential (Santopinto et al.,1995)which is attractive for small separations.

$$V_{HYC.}(r) = -\tau r^{-1} \tag{5}$$

where $V_o, k, and \tau$ are constants. the quark potential V , is supposed to depend on hyper radius r only, that is to be hypercentral. Therefore $V=V(r)$ is in general a three body potential, since hyper radius r depends on the coordinates of all three quarks. The following analytical solution of potential is employed (Salehi and Rajabi,2009). The energy eigen values for the mode $v=0$ and grand angular momentum are given as follows.

$$E_{0\gamma} = (2\gamma + 6) \frac{\omega}{2} - \frac{2mr^2}{(2\gamma + 5)^2} \tag{6}$$

and the ground state normalized eigenfunctions are given as

$$\Psi_{0\gamma} = N_\gamma x^{-\frac{5}{2}} \Phi_{0\gamma} = N_\gamma x^\gamma \exp\left(-\frac{m\omega}{2} x^2 - \frac{2mr}{(2\gamma + 5)} x^2\right) \tag{7}$$

Similarly we can continue for other modes ($1,2,3,\dots$).

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The hypercentral constituent quark model is fairly good for non strange baryon spectrum but in some cases splitting within the SU(Giannini et al., 2005) baryon multiplets is provided by Gursev - Redicati mass formula (Gursev and Radicati, 1994).

$$M = M_0 + CS(S + 1) + DY + E[T(T + 1) - \frac{1}{4}Y^2] \quad (8)$$

where M_0 is the average energy value of the SU (Giannini et al., 2005) multiplets, S is the total spin, Y is the hypercharge and T is the total isospin of baryons. Eq.(8) can be written in terms of Casimir operators in the following way

$$M = M_0 + CC_2[SU_s(2) + DC_1[U_Y(1)] + E[C_2[SU_f(2) - \frac{1}{4}(C_1[U_Y(1)]^2)] \quad (9)$$

where $C_2[SU_s(2)]$ and $C_2[SU_f(2)]$ are the SU(2) (quadratic) Casimir operators for spin and isospin, respectively, and $C_1[U_Y(1)]$ is the Casimir for U(1) subgroup generated by the hypercharge Y. Giannini et al. 2005 considered a dynamical spin-flavor symmetry $SU_{SF}(6)$ (Bijker et al., 2004) and describe the symmetry breaking mechanism by generalizing Eq.(9) as

$$M = M_0 + AC_2[SU_{SF}(6) + BC_2[SU_f(3)] + CC_2[SU_s(2)] + DC_1[U_Y(1)] + E[C_2[SU_f(2) - \frac{1}{4}(C_1[U_Y(1)]^2)] \quad (10)$$

Table 2: Mass Spectrum of N And Δ Resonances

| Baryon | Status | State | M _{PDG} [13] | M _{theor.} [11] | M _{our calc.} |
|------------|--------|---|-----------------------|--------------------------|------------------------|
| N(938)P11 | **** | ² 8 _{1/2} [56,0] | 938 | 938.0 | 938.7 |
| N(1440)P11 | **** | ² 8 _{1/2} [56,0] | 1430-1470 | 1448.7 | 1467.5 |
| N(1535)S11 | **** | ² 8 _{1/2} [70,1] | 1520-1555 | 1543.8 | 1544.9 |
| N(1650)S11 | **** | ⁴ 8 _{1/2} [70,1] | 1640-1680 | 1658.6 | 1642.7 |
| N(1675)D15 | **** | ⁴ 8 _{5/2} [70,1] | 1670-1685 | 1658.6 | 1642.1 |
| N(1700)D13 | *** | ⁴ 8 _{3/2} [70,1] | 1650-1750 | 1658.6 | 1635.8 |
| N(1710)P11 | *** | ² 8 _{1/2} [56,0] | 1680-1740 | 1795.4 | 1780.1 |
| (1232)P33 | **** | ⁴ 10 _{3/2} [56,0] | 1230-1234 | 1232.0 | 1229.9 |
| (1600)P33 | *** | ⁴ 10 _{3/2} [56,0] | 1550-1700 | 1683.0 | 1706.7 |
| (1620)S31 | **** | ² 10 _{1/2} [70,1] | 1615-1675 | 1722.8 | 1710.5 |
| (1700)D33 | **** | ² 10 _{3/2} [70,1] | 1630-1770 | 1722.8 | 1710.5 |
| (1905)P31 | **** | ⁴ 10 _{5/2} [56,2 ²] | 1870-1920 | 1945.4 | 1878.1 |
| (1910)P33 | **** | ⁴ 10 _{1/2} [56,2 ²] | 1870-1920 | 1945.4 | 1878.1 |
| (1920)P33 | *** | ⁴ 10 _{3/2} [56,0] | 1900-1970 | 2089.4 | 2037.3 |

It is a conditional generalized Gursev - Redicati mass formula. Therefore, the non strange baryon masses are obtained by three quark masses and the eigen energies (E_{τ}) of the radial Schrödinger equation with the expectation values of H_{GR} as

$$M = \sum m_q + E_{\nu_f} + \langle H_{GR} \rangle \quad (11)$$

where m_q are the constituent quark masses. for calculation the constituent quark masses are assumed (m_u m_d m_s). In equation no.11, H_{GR} is in the following form.

$$H_{GR} = AC_2[SU_{SF}(6) + BC_2[SU_f(3)] + CC_2[SU_s(2)] + DC_1[U_Y(1)] + E[C_2[SU_f(2) - \frac{1}{4}(C_1[U_Y(1)]^2)] \quad (12)$$

The expectation values of H_{GR} , ($\langle H_{GR} \rangle$), is completely identified by the expectation values of Casimir operators(Bijker et al.,2004). For calculating the baryon masses, we choose a limited no. of well known strange parameters.

Table 1: Parameters in The Model

| Parameter | Value |
|-----------|-----------------------|
| A | -7.94 MeV |
| B | 2.23 MeV |
| C | 37.9 |
| D | -195.9 MeV |
| E | 52.3 MeV |
| m | 283 MeV |
| τ | 4.59 fm ⁻¹ |
| ω | 0.46 |
| V_0 | 0.1 MeV |

RESULTS AND DISCUSSION

In this paper we have done a comprehensive study of masses of N and Δ baryon resonances. we have employed a simple hypercentral constituent quark model and the generalized Gursesey-Radicati mass formula. The computed results for few N and Δ states are listed in Table, 2. The overall good description of the spectrum which we obtain by the combination of potentials shows that our model can also be used to give a fair description of the energy of excited multiplet up to 2 GeV. Although our results of the resonances $N(1440)$ and other excited states are in good agreement with the experimental results but while comparing with the other theoretical models some problems are faced in the reproduction of the experimental masses of some baryons and in particular for the $(1920)P_{33}$ and the $N(1710)P_{11}$ resonances that's come out degenerate and above the experimental values. The better agreement can be obtained to increase of the spatial excitation which dependence on the decrease of SU(Giannini et al.,2005) breaking part by including some other factors like a delta or Gaussian factor.

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