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FRW COSMOLOGICAL MODEL OF THE UNIVERSE AND DECELERATION PARAMETER

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ABSTRACT

Constracted homogeneous and isotropic FRW Cosmological model with constant deceleration parameter and time dependent gravitational and cosmological constant. The Einstein's field equations have been solved by applying a variation law for generalized Hubble's parameter in FRW space- time. The variation law for Hubble's parameter generates two types of solutions for the average scale factor; one is of power law type and other is of the exponential form.

KEY WORDS: Cosmology, FRW Model, Hubble parameter, constant deceleration parameter, variable

The Einstein's field equations are a coupled system of highly nonlinear differential equations and we seek physical solutions to the field equations for applications in cosmology and astrophysics. In order to solve the field equations we normally assume a form for the matter content or suppose that spacetime admits Killing vector symmetries Kramer and Schmutzer (1980). Solutions to the field equations may also be generated by applying a law of variation for Hubble's parameter which was proposed by Berman and Cimento (1983). It is interesting to observe that this law yields a constant value for the deceleration parameter. Forms for the deceleration parameter which are variable have been investigated by Beesham (1993). The variation of Hubble's law assumed is not inconsistent with observation and has the advantage of providing simple functional forms of the scale factor. In earlier literature cosmological models with constant deceleration parameter have been studied by Singh and Baghel (2009) , Berman and Gomid (1988) , Maharaj and Naidoo (1993), and Mohanti et al. (2009) whereas Pradhan et al. (2006) have been studied FRW model with variable deceleration parameter.The case of perfect fluid Robertson-Walker spacetime with variable gravitational and cosmological constants has been pursued by Berman (1991). A treatment may also be performed in alternate theories of gravity; for example Berman and Gomid (1988) consider applications to the Price Hoyle and Brans Dicke theory.

In this paper we have investigate FRW cosmological model by obtaining solutions to the field equations, with variable gravitational and cosmological constants .The scale factors is explicitly determined by the law of variation for Hubble's parameter.

Equation Gverning The Cosmological Models

In standerd coordinates $(x^a) = (t, r, \theta, \Phi)$ the Robertson-Walker line element has the form

$$ds^2 = dt^2 - S^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\emptyset^2\right)\right] \quad (1)$$

Where S(t) is the cosmic scale factor. Without loss of generality curvature index K takes on only three values 0, 1 or -1 such that k = 0 for the flat model, k = 1 for a closed model and k = -1 for an open model. This describes the space type geometry of the universe on large scales in comoving coordinates that is the content of the universe are on average at rest in the coordinate system (r, θ, Φ). The time "t" is the proper time for the particles which are in this case clusters of galaxies.

Let us consider a particle and the origin r=0 and another particle at "r" them the proper distance "D" between two particles at a time "t" given by

$$D = S(t)\int_0^{r}\frac{dr}{\sqrt{1-kr^2}} = \{R\sinh^{-1} r \qquad (k=-1)\}$$

$$Rr \qquad (k = 0)$$

$$R\sin^{-1} r \qquad (k = 1) \quad (2)$$

Therefore the proper distance 'D' is proportional to the scale S(t). The proper velocity v of particle at ' r ' at the origin is obtained by differentiating D w.r.to "t" relating that "r" remains constant because it is coming co-ordinate. Thus we have

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$$v = D = S(t)\int_0^S \frac{dr}{\sqrt{1-kr^2}} = \frac{\dot{S}}{S} D. \tag{3}$$

where a dot on the symbol means differentiation with respect to t. This tells us that at any time t the speed "v" is proportional to the proper distance D. Comparing this result with the Hubble's law v = HD

we get $H = \frac{\dot{S}}{S}$ (4)

In fact, we cannot stretch the measuring tape between the particles, therefore the proper distance "D" is not measurable quantity. However, the great advantage of the relativistic formulation is that it gives relationship between quantities such as red shifts apparent magnitudes, number counts etc. which can be measured. The important point is that they all involve only the scale factor S(t).

Therefore, the scale factor has been treated as a measure of distance. In a similar way and are treated as the measures of the velocity and the acceleration of the fundamental particles relative to the origin that is in the space time metric Rahman (1990), we consider the commoving coordinate r dimensionless and the scale factor S(t) as having the dimension of length .

By taking large scale viewpoint the energy momentum tensor of the content of the universe takes the same form as for a perfect fluid distribution of matter.

$$T^{ij} = (p+\rho)u^i u^j - p\, g^{ij} \tag{5}$$

where p is the proper pressure, ρ is the proper density and is the four velocity of the fluid particles which are in this case cluster of galaxies. Since the particle is at rest in the coordinate system (r, θ, Φ) we have

$$u^i = (0,0,0,1) \tag{6}$$

We consider the Einstein field equation as

$$R_{ij} - \tfrac{1}{2}Rg_{ij} + \Lambda g_{ij} = -8\pi G T_{ij} \tag{7}$$

and the conservation law as $T^{ij}_{;j} = 0$ (8)

which on applying to (7), we get

$$\Lambda_{ji}\, g^i_j + 8\pi G T^i_j = 0 \tag{9}$$

Substituting the values from (1) , (5) , and (6) in (7) we get the following set of equations

$$2\frac{\ddot{S}}{S} + \frac{(\dot{S}^2+k)}{S^2} = -8\pi gp + \Lambda \tag{10}$$

$$3\frac{\dot{S}^2}{S^2} + \frac{3k}{S^2} = 8\pi gp + \Lambda \tag{11}$$

Differentiating equation (11) with respect to "t" we get

$$3\frac{\dot{S}}{S}\left[2\frac{\ddot{S}}{S} - \frac{2S^2}{S^2} - \frac{2k}{S^2}\right] - \Lambda = 8\pi\dot{G}\rho + 8\pi G\dot{\rho} \tag{12}$$

Subtracting equation (10) from (11) we get

$$\left[-2\frac{\ddot{S}}{S} + \frac{2\ddot{S}^2}{S^2} + \frac{2k}{S^2}\right] = 8\pi G(p+\rho) \tag{13}$$

Inserting equation (13) in equation (12) we get

$$\dot{G}\rho + G\dot{\rho} + 3Gp\frac{\dot{S}}{S} + 3G\rho\frac{\dot{S}}{S} + \frac{\Lambda}{8\pi} = 0$$

Rearranging above equation after multiplying byR^3, we obtain

$$G\left[\frac{d(\rho s^3)}{dt} + 3pS^2\dot{S}\right] + S3\left[\dot{G}\rho + \frac{\Lambda}{8\pi}\right] = 0 \tag{14}$$

Equation (8) when simplified with the help of the equations (1) , (5) and (6) we get the conservation law as

$$\frac{d(\rho s3)}{dt} + 3pS^2\dot{S} = 0 \tag{15}$$

From equation (14) and (15) , we get

$$G\rho + \frac{\Lambda}{8\pi} = 0 \tag{16}$$

Equation (10) is multiplying by 3 and then adding with (11) we obtain

$$3\ddot{S} = -4\pi GS(3p+\rho) + \Lambda S \tag{17}$$

Equation (11), (15), (16) and (17) are the fundamental equations governing a homogeneous and isotropic cosmological model of the universe. Equation (11), (15) and (17) are same as the corresponding equation in general relativity, but here we have additional equation (16) due to the variation of G and Λ. We consider the dependence of the pressure in a homogeneous and isotropic model of the universe as

$$P = \omega\rho \tag{18}$$

Where $\omega \in [0, 1]$ (19)

The equality sign [ω = 0] in (19) implies that the model is pressure less.

Eqs. (15) and (18) together give the dependence of ρ on S, but this is not sufficient to get the dependence of S on t from eqs. (11). For this we need the dependence of G and Λ on S. Also knowing the dependence of ρ on S. Eqs. (16) is integrable when we have an additional equation relating G and Λ as

$$G\rho = \frac{\eta\Lambda}{8\pi} \quad (20)$$

And thus , the fundamental equation governing the model of the universe are (11) , (15) , (16) (17), (18) and (20). Now from equation (15) and (18), we get

$$\rho = \alpha\, S^{-3(1+\omega)} \quad (21)$$

Then from equation (16), (20) and (21) and putting a=1, we get

$$G = S^{\frac{3(1+\omega)}{(1+\eta)}} \quad (22)$$

$$\Lambda = S^{\frac{-3\eta(1+\omega)}{(1+\eta)}} \quad (23)$$

To solve completely the system of equation (10) and (11) we assume that the variation of Hubble parameter is given by the equation

$$H = l S^{-n} \quad (24)$$

From equation (4) and (24) we get $S^{n-1}\dot{S} = l$

Integrating above equation we obtain the law of average scale factor "S" as

$$S = (nlt + c_1)^{\frac{1}{n}} \text{ ; for } n \neq 0 \quad (25)$$

$$S = c_2 e^{lt} \text{ ; for } n = 0 \quad (26)$$

Where C_1 and C_2 are constants of integration. Thus the law (24) provides two type of the expansion in the (i) power law and (ii) exponential law

Case-I When $n \neq 0$ I e. model for power law expansion :

In this case the metric (1) takes the form

$$ds^2 = dt^2 - (nlt + c_1)^{\frac{2}{n}}\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\emptyset^2\right] \quad (27)$$

Using equation (25) in equation (4) the Hubble parameter H and expansion scalar θ are given by

$$H = (nlt + c_1)^{-1} \quad (28)$$

$$\theta = 3H = 3(nlt + c_1)^{-1} \quad (29)$$

The deceleration parameter q is given by

$$q = n-1 \quad (30)$$

this shows that in this case the deceleration parameter is constant. Using equation (25) in equations (18), (21), (22) and (23) pressure p, energy density ρ gravitational constant G and cosmological constant Λ are given by

$$p = \omega\, (nlt + c_1)^{\frac{-3(1+\omega)}{n}} \quad (31)$$

$$\rho = (nlt + c_1)^{\frac{-3(1+\omega)}{n}} \quad (32)$$

$$G = (nlt + c_1)^{\frac{3(1+\omega)}{n(1+\eta)}} \quad (33)$$

$$\Lambda = (nlt + c_1)^{\frac{-3\eta(1+\omega)}{n(1+\eta)}} \quad (34)$$

Discussion

The deceleration parameter q for the model (27) is constant. The energy density and pressure for this model are given by equation (31) and (32). It is evident that ρ and $p \to 0$ as $t \to \infty$ whereas ρ and p both are approach infinite at $t = -C_1/nl$. Cosmological constant Λ is decrease as cosmic time t increase whereas gravitational constant G is increases as t increases.

Case - II When n = 0 i.e. Model for exponential law expansion:

In this case the metric (1) takes the form

$$ds^2 = dt^2 - (c_2 e^{lt})^2\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\emptyset^2\right] \quad (35)$$

The deceleration parameter q is given by

$$q = -1 \quad (36)$$

Using equation (26) in equations (18), (21), (22) and (23) pressure p, energy density ρ gravitational constant G and cosmological constant Λ are given by

$$p = \omega(c_2)^{-3(1+\omega)}\, e^{-3lt(1+\omega)} \quad (37)$$

$$\rho = (c_2)^{-3(1+\omega)}\, e^{-3lt(1+\omega)} \quad (38)$$

$$G = (c_2)^{\frac{3(1+\omega)}{(1+\eta)}}\, e^{\left\{\frac{3(1+\omega)lt}{(1+\eta)}\right\}} \quad (39)$$

$$\Lambda = (c_2)^{\frac{-3\eta(1+\omega)}{(1+\eta)}}\, e^{\left\{\frac{-3\eta(1+\omega)lt}{(1+\eta)}\right\}} \quad (40)$$

The Hubble parameter H and expansion scalar θ are given by

$$H = l \quad (41)$$

$$\theta = 3l \quad (42)$$

The energy density ρ, pressure p, and cosmological constant Λ all are approach to zero as $t \to \infty$ and gravitational constant $G \to \infty$ as $t \to \infty$.Cosmological constant $\Lambda \to \infty$ as $t \to -\infty$.For this model, deceleration parameter q = -1. Therefore the model represents an accelerating universe. As $t \to -\infty$, parameters p and ρ becomes infinite. The expansion scalar θ is constant through out the expansion. Therefore the model represents uniform expansion.

CONCLUSION

We have constructed homogeneous and isotropic FRW cosmological model with time varying cosmological and gravitational term by assuming a variation law for the Hubble's parameter that yields a constant value of the deceleration parameter. Universe models for $n \neq 0$ and $n = 0$ have been derived. When $n = 0$, we obtain for the model (35) $q = -1$ and $dH/dt=0$ which implies the gratest value of the Hubble parameter and the fastest rate expansion of the universe. Thus, this model may represent the the inflationary era in the early universe and the very late times of the universe. From equation (30) it is clear that the universe accelerates for $0 \leq n < 1$ decelerates for $n > 1$ and exponds with constant velocity for $n = 1$.

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REFERENCES

Berman M.S. and Cimento N.B., 1983. Special law of variation for Hubble parameter. Italy **74** : 182 6.

Berman M.S. and Gomid F.M., 1988. Cosmological model with constant deceleration parameter. Gen. Rel. Gravit., **20** : 191-198.

Berman M.S., 1991. Cosmological model with variable gravitational and cosmological constants. Gen. Rel. Gravi., **23** : 465-469.

Beesham A.K., 1993. Cosmological model with variable cosmological term and bulk viscous model. Phys. Rev., **48** : 3539-3543.

Kramer D. and Schmutzer E., 1980. Exact solutions of Einstein's Field Equation. Cambridge University Press, Cambridge, New York .

Maharaj S.D. and Naidoo R.; 1993. Solution to the field equation and the deceleration parameter. Astrophys. Space Sci., **208** : 261-276.

Mohanti G., Samanta G.C. and Mahanta K.L., 2009. Kaluza-Klein FRW Cosmological model in Lyra manifold. Theoret. Appl. Mech., **36** : 157-166.

Pradhan A., Shahi J.P. and Singh C.B., 2006. Cosmological model of the universe with variable deceleration parameter in Lyra's manifold. Brazilian J. of Phys., **36** : 1227-1231.

Rahman A., 1990. A critical density cosmological model with varying gravitational and cosmological constant. Gen. Rel. Gravit., **22** : 655-657

Singh J.P. and Baghel P.S., 2009. Bianchi type V bulk viscous cosmological model with time dependent Ë term. Elec. J. of Theor. Phys., **6** : 85-96.