Indian J.Sci.Res. 14 (1): 326-333, 2017
ISSN: 0976-2876 (Print)
ISSN: 2250-0138 (Online)

FUZZY INVENTORY MODEL FOR DETERIORATION ITEMS THROUGH JUST IN TIME WITH SHORTAGES ALLOWED

J. JAYANTHI[a1] AND M. MARAGATHAM[b]

[a]Department of Mathematics, Periyar Maniammai University, Thanjavur, India
[b]P.G. & Research Department of Mathematics, Periyar EVR College, Tiruchirappalli, India

ABSTRACT

This paper derives an inventory model for deterioration items through JIT with shortages has been considered in a fuzzy environment. To determine the optimal total cost and the optimal order quantity for the proposed inventory model. In order to accomplish this purpose we have been introduced the trapezoidal fuzzy numbers. The working out of economic order quantity (EOQ) is conceded out through defuzzification process by using signed distance method. The signed distance method is more pertinent than the other methods of defuzzification. To illustrate the results of the proposed model, we have given two model examples and presented the computational results. Sensitivity for this model is also calculated, which shows a linear relation among demand, EOQ and total cost. This anticipated approach is simple and gives a better result in comparatively less computational work.

KEYWORDS: Just – In –Time; Quality Assurance; Deteriorating Items; Trapezoidal Fuzzy Numbers; Defuzzification; Signed Distance Method.

Inventory plays an extremely important role in an international economy. An inventory may be in the form of raw materials, fulfilled parts, impartial parts, semi – finished goods or finished goods. There are four types of inventory items namely (1) Deterioration items (2) Obsolescence items (3) Demolish or Pilferage items (4) No Obsolescence / Deterioration items. Deterioration items refers to the items that become decomposed, smashed, expired, illogical, depreciation or physical characteristics of a material due to flawed packaging or abnormal storage conditions. Obsolescence items are referred to the archaic products lose their value because of hasty changes of equipment. Damage or Pilferage items are referred to the crime of stealing involving employees who steal items from their consign of service, in particular in a manufacturing plant. No Obsolescence / Deterioration items are referred to the life cycle of some goods is imprecise in nature.

Slack et al. refer Voss's definition of JIT: "a disciplined approach to improving overall productivity and eliminating waste. It provides for the cost-effective production and delivery of only the necessary quantity of parts at the right quality, at the right time and place, while using a minimum amount of facilities, equipment, materials and human resources. JIT is dependent on the balance between the supplier's flexibility and the user's flexibility. It is accomplished through the application of elements which require total employee involvement and team-work. A key philosophy of JIT is simplification".

The widespread adoption of just-in-time (JIT) inventory principles undoubtedly makes production operations more efficient, cost effective and customer responsive. Companies effectively implementing JIT principles have substantial competitive advantages over competitors that have not. The trick is figuring out how to apply JIT principles to gain competitive advantages in your specific industry and business situation. The basic premise of JIT is to have just the right amount of inventory, whether raw materials or finished goods, available to meet the demands of your production process and the demands of your end customers. No more, nor less.

This paper considers a simple and convenient situation and derives the minimum optimal solution with deteriorating items which integrates inventory and eminence affirmation in a JIT . Shortages are allowed and lead time is zero.

This model was developed by Harris. Wilson aroused interest in the EOQ model in academics and industries. Later, Hadley et al analyzed many inventory systems. In certain situations, uncertainties are due to fuzziness, primarily introduced by Zadeh, is applicable. In 1970, Zadeh et al proposed some strategies for decision making in fuzzy environment. Jain worked on decision making in the presence of fuzzy variables. Kacpryzk et al discussed some long-term inventory policy-making through fuzzy-decision making models.Wide applications of fuzzy set theory can be found in Zimmerman, and Park. Urgeletti treated EOQ model in fuzzy sense, and used triangular fuzzy number. Chan and Wang used trapezoidal fuzzy number to fuzzify the order cost, inventory cost, and backorder cost in the total cost of inventory model without backorder.

Vujosevic et al used trapezoidal fuzzy number to fuzzify the order cost in the total cost of inventory model with backorder. Further, in a series of papers, Yao

[1]Corresponding author

et al., considered the fuzzified problems for the inventory with or without backorder models. Syed & Aziz used trapezoidal fuzzy number. De and Rawat, proposed an EOQ model without shortage cost by using triangular fuzzy number. The total cost has been computed by using signed distance method. In, Dutta and Pavan Kumar derived a fuzzy inventory model without shortage using trapezoidal fuzzy number with sensitivity analysis.

Here we develop a fuzzy inventory model with shortage using trapezoidal fuzzy number with sensitivity analysis using the signed distance method for defuzzification. The annual total cost is calculated as a function of five variables (ie) holding cost, set up cost, shortage cost, screening cost and reworking cost.

METHODOLOGY

Fuzzy Numbers

Any fuzzy subset of the real line R, whose membership function μ_A satisfied the following conditions, is a generalized fuzzy number $\widetilde{A}$.

(i) μ_A is a continuous mapping from R to the closed interval

[0, 1].

(ii) $\mu_A = 0, -\infty < x \le a_1$,

(iii) $\mu_A =$ L(x) is strictly increasing on $[a_1, a_2]$

(iv) $\mu_A = w_A, a_2 \le x \le a_3$

(v) $\mu_A =$ R(x) is strictly decreasing on $[a_3, a_4]$

(vi) $\mu_A = 0, a_4 \le x < \infty$

where $0 < w_A \le 1$ and a_1, a_2, a_3 and a_4 are real numbers. Also this type of generalized fuzzy number be denoted as $\widetilde{A} = (a_1, a_2, a_3, a_4 : w_A)_{LR}$; When $w_A = 1$, it can be simplified as $\widetilde{A} = (a_1, a_2, a_3, a_4)_{LR}$

Trapezoidal Fuzzy Number

A trapezoidal fuzzy number $\tilde{A}$ = (a, b, c, d) is represented with membership function $\mu_{\tilde{A}}$ as:

$$\mu_{\tilde{A}}(x) = \begin{cases} L(x) = \frac{x-a}{b-a}, & when\ a \le x \le b; \\ 1, & when\ b \le x \le c; \\ R(x) = \frac{d-x}{d-c}, & when\ c \le x \le d; \\ 0, & otherwise \end{cases}$$

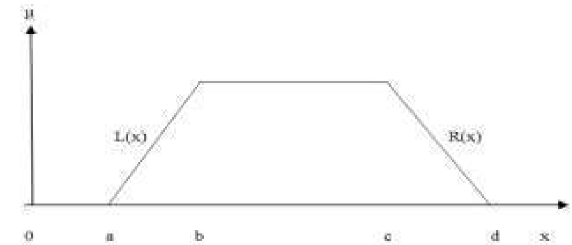

Figure 1: Trapezoidal Fuzzy Number

The Function Principle

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers, then arithmetical operations are defined as:

1. $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
2. $\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$
3. $\tilde{A} \ominus \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$
4. $\tilde{A} \oslash \tilde{B} = (\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1})$
5. $\alpha \otimes \tilde{A} = \begin{cases} (\alpha a1, \alpha a2, \alpha a3, \alpha a4), \alpha \ge 0 \\ (\alpha a4, \alpha a3, \alpha a2, \alpha a1), \alpha < 0 \end{cases}$

Signed Distance Method

Let $\tilde{A}$ be a fuzzy set defined on R. Then the signed distance of $\tilde{A}$ is defined as:

$$d(\tilde{A}, 0) = \frac{1}{2}\int_0^1 [A_L(\alpha) + A_R(\alpha)] d\alpha$$

where $A_\alpha = [A_L(\alpha), A_R(\alpha)]$

$= [a + (b - a)\alpha, d - (d - c)\alpha], \alpha \in [0, 1]$,

is α − cut of fuzzy set $\tilde{A}$, which is a close interval.

NOTATIONS AND ASSUMPTIONS

The mathematical model of this paper is developed on the basis of the following notations and assumptions.

Notations

c : holding cost per unit quantity per unit time

s : set up or ordering cost per order

r : shortage cost or stock out cost per unit quantity per unit time

q : order quantity per cycle

t : scheduling time period

z : order level

R : total demand over the planning time period [0, t]

S : screening cost per unit

β : reworking cost per unit

θ : percentage of defective items

TC : total annual cost for the period[0, t]

$\tilde{c}$: fuzzy holding cost per unit quantity per unit time

$\tilde{r}$: fuzzy shortage cost per unit quantity per unit time

$\tilde{s}$: fuzzy set up or ordering cost per order

$\widetilde{TC}$: fuzzy total cost for the period [0, t]

F (q) : de-fuzzifyed total cost for [0, t]

F(q)* : mimimum de-fuzzifyed total cost for [0, t]

q_d* : optimal order quantity

Assumptions

- Demand rate is uniform and finite.
- Lead time is zero.
- Shortages are allowed.
- Inventory is continuously reviewed.
- Screening cost and reworking cost is constant.
- Holding cost , setup cost and shortage cost are taken as a trapezoidal fuzzy numbers.
- Only a single order will be produced at the beginning of each cycle and the entire lot is delivered in one batch.
- c be the inventory carrying cost per unit quantity per unit time, r be the shortage cost per unit quantity per unit time & s be the ordering cost per order, known and constant.
- q is the lot-size per cycle whereas z is the initial inventory level after fulfilling the back-logged quantity of previous cycle and q – z be the maximum shortage level.
- t is the cycle length or scheduling period.

MODEL FORMULATION

Proposed Inventory Model In Crisp Sense

From the above notations and assumptions, we obtain the total annual cost for the inventory model for deterioration items through JIT with shortages, in crisp environment.

The total cost for the period (0, t) is given by,

TC(q) = carrying cost + set up cost + shortage cost + screening cost + reworking cost

$$TC(q) = \frac{cz^2}{2Rt} + \frac{s}{t} + \frac{r(Rt-z)^2}{2Rt} + SRt + \beta\theta Rt \quad (1)$$

$$TC(q) = \frac{cz^2}{2q} + \frac{sR}{q} + \frac{r(q-z)^2}{2q} + Sq + \beta\theta q$$

where $z = \frac{rRt}{c+r}$ and $q = Rt$

$$TC(q) = \frac{crq}{2(c+r)} + \frac{sR}{q} + Sq + \beta\theta q \quad (2)$$

Partially differentiate equation (1) with respect to t, we get

$$\frac{\partial TC}{\partial t} = -\frac{cz^2}{2Rt^2} - \frac{s}{t^2} + \frac{rR}{2} - \frac{rz^2}{2Rt^2} + SR + \beta\theta R \quad (3)$$

The optimum $q*$ and $TC*$can be obtained by equating the first partial derivative w.r.t to 't' of TC to zero.

(i.e) $\frac{\partial Tc}{\partial t} = 0$ gives

$$t* = \sqrt{\frac{2s(c+r)}{crR + 2SR(c+r) + 2\beta\theta R(c+r)}}$$

(optimum period) (4)

Optimal order quantity

$q*$=

$$Rt* = R\sqrt{\frac{2s(c+r)}{crR + 2SR(c+r) + 2\beta\theta R(c+r)}} \quad (5)$$

$$\Rightarrow q* = \sqrt{\frac{2Rs(c+r)}{cr + 2S(c+r) + 2\beta\theta(c+r)}} \quad (6)$$

Minimum total cost

$$TC* = \sqrt{2Rs\left[\frac{cr}{c+r} + 2S + 2\beta\theta\right]} \quad (7)$$

Diagrammatic Representation:

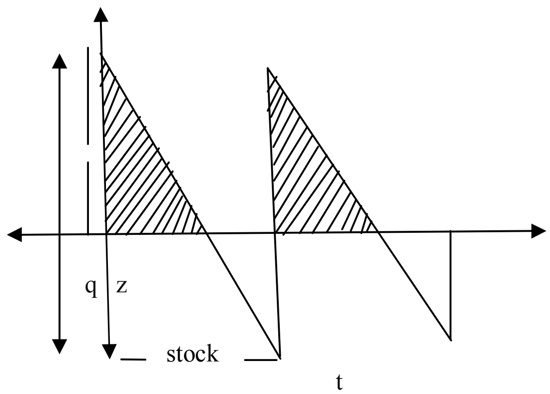

Figure 2 : Variation of quantity q with respect to time t

Proposed Inventory Model In Fuzzy Sense

Here, we consider the model in fuzzy environment. Since the holding cost , set up cost and shortage cost are fuzzy in nature, we represent them by trapezoidal fuzzy numbers.

Let $\tilde{c}$: fuzzy carrying or holding cost per unit quantity per unit time

$\tilde{s}$: fuzzy set up or ordering cost per order

$\tilde{r}$: fuzzy shortage cost or stock out cost per unit quantity per unit time

The total demand is consider as constant. Now we fuzzify total cost given in (2), the fuzzy total cost is given by:

$$\widetilde{TC}(q) = \frac{\tilde{c}\,\tilde{r}\,q}{2(\tilde{c}+\tilde{r})} + \frac{\tilde{s}R}{q} + Sq + \beta\theta q \quad (8)$$

To pertain signed distance method to defuzzify the fuzzy total annual cost, and then acquire the optimal order quantity (q_d*****) by using simple calculus technique.

Suppose $\tilde{c} = (c_1, c_2, c_3, c_4)$, $\tilde{s} = (s_1, s_2, s_3, s_4)$ and $\tilde{r} = (r_1, r_2, r_3, r_4)$ are fuzzy trapezoidal numbers, in LR form, where $0 < r < s < c$ and $r_1, r_2, r_3, r_4, s_1, s_2, s_3, s_4, c_1, c_2, c_3$, and c_4 are known positive numbers.

From (8), we have:

$$\widetilde{TC}(q) = \left[\frac{\tilde{c}\otimes\tilde{r}\otimes q}{2\otimes(\tilde{c}\oplus\tilde{r})}\right]\oplus\left[\frac{\tilde{s}\otimes R}{q}\right]\oplus[S\otimes q]\oplus[\beta\otimes\theta\otimes q] \quad (9)$$

$$= \left[\frac{(c_1,c_2,c_3,c_4)\otimes(r_1,r_2,r_3,r_4)\otimes q}{2\otimes((c_1,c_2,c_3,c_4)\oplus(r_1,r_2,r_3,r_4))}\right]\oplus\left[\frac{s_1,s_2,s_3,s_4\otimes R}{q}\right]\oplus[S\otimes q]\oplus[\beta\otimes\theta\otimes q]$$

$$= \left[\frac{(c_1r_1q, c_2r_2q, c_3r_3q, c_4r_4q)}{2\otimes((c_1+r_1, c_2+r_2, c_3+r_3, c_4+r_4))}\right]\oplus\left[\frac{s_1R, s_2R, s_3R, s_4R}{q}\right]\oplus[Sq]\oplus[\beta\theta q]$$

$$= \left[\frac{(c_1r_1q, c_2r_2q, c_3r_3q, c_4r_4q)}{2(c_1+r_1),\ 2(c_2+r_2),\ 2(c_3+r_3),\ 2(c_4+r_4)}\right]\oplus\left[\frac{s_1R, s_2R, s_3R, s_4R}{q}\right]\oplus[Sq]\oplus[\beta\theta q]$$

$$= \left[\frac{c_1r_1q}{2(c_1+r_1)}, \frac{c_2r_2q}{2(c_2+r_2)}, \frac{c_3r_3q}{2(c_3+r_3)}, \frac{c_4r_4q}{2(c_4+r_4)}\right]\oplus\left[\frac{s_1R, s_2R, s_3R, s_4R}{q}\right]\oplus[Sq]\oplus[\beta\theta q]$$

$$\widetilde{TC} = \left[\begin{array}{l}\frac{c_1r_1q}{2(c_1+r_1)} + \frac{s_1R}{q} + Sq + \beta\theta q, \frac{c_2r_2q}{2(c_2+r_2)} + \frac{s_2R}{q} \\ + Sq + \beta\theta q, \frac{c_3r_3q}{2(c_3+r_3)} + \frac{s_3R}{q} + Sq + \beta\theta q, \\ \frac{c_4r_4q}{2(c_4+r_4)} + \frac{s_4R}{q} + Sq + \beta\theta q\end{array}\right] \quad (10)$$

$= (a, b, c, d)$ (say)

Now $A_L(\alpha) = a + (b-a)\alpha$

$$= \left[\frac{c_1r_1q}{2(c_1+r_1)} + \frac{s_1R}{q} + Sq + \beta\theta q\right] + \left[\left[\frac{c_2r_2}{2(c_2+r_2)} - \frac{c_1r_1}{2(c_1+r_1)}\right]q + \frac{(s_2-s_1)R}{q}\right]\alpha \quad (11)$$

and $A_R(\alpha) = d - (d-c)\alpha$

$$= \left[\frac{c_4r_4q}{2(c_4+r_4)} + \frac{s_4R}{q} + Sq + \beta\theta q\right] - \left[\left[\frac{c_4r_4}{2(c_4+r_4)} - \frac{c_3r_3}{2(c_3+r_3)}\right]q + \frac{(s_4-s_3)R}{q}\right]\alpha \quad (12)$$

Defuzzifying $\widetilde{TC}$ in (10) by using signed distance method, we have:

$$d(\widetilde{TC}) = \frac{1}{2}\int_0^1 [A_L(\alpha) + A_R(\alpha)]\,d\alpha$$

$$= \frac{1}{2}\int_0^1 A_L(\alpha)\,d\alpha + \frac{1}{2}\int_0^1 A_R(\alpha)\,d\alpha$$

$$= \frac{1}{2}\int_0^1 \left\{ \begin{array}{l} \left[\frac{c_1 r_1 q}{2(c_1+r_1)} + \frac{s_1 R}{q} + Sq + \beta\theta q\right] + \\ \left[\left(\frac{c_2 r_2}{2(c_2+r_2)} - \frac{c_1 r_1}{2(c_1+r_1)}\right)q + \frac{(s_2-s_1)R}{q}\right]\alpha \end{array} \right\} d\alpha$$

$$+ \frac{1}{2}\int_0^1 \left\{ \begin{array}{l} \left[\frac{c_4 r_4 q}{2(c_4+r_4)} + \frac{s_4 R}{q} + Sq + \beta\theta q\right] - \\ \left[\left(\frac{c_4 r_4}{2(c_4+r_4)} - \frac{c_3 r_3}{2(c_3+r_3)}\right)q + \frac{(s_4-s_3)R}{q}\right]\alpha \end{array} \right\} d\alpha$$

$$= \frac{1}{2}\left[\frac{c_1 r_1 q}{2(c_1+r_1)} + \frac{s_1 R}{q} + Sq + \beta\theta q\right][\alpha]_0^1 + \frac{1}{2}\left[\left(\frac{c_2 r_2}{2(c_2+r_2)} - \frac{c_1 r_1}{2(c_1+r_1)}\right)q + \frac{(s_2-s_1)R}{q}\right]\left[\frac{\alpha^2}{2}\right]_0^1$$

$$+ \frac{1}{2}\left[\frac{c_4 r_4 q}{2(c_4+r_4)} + \frac{s_4 R}{q} + Sq + \beta\theta q\right][\alpha]_0^1 - \frac{1}{2}\left[\left(\frac{c_4 r_4}{2(c_4+r_4)} - \frac{c_3 r_3}{2(c_3+r_3)}\right)q + \frac{(s_4-s_3)R}{q}\right]\left[\frac{\alpha^2}{2}\right]_0^1$$

$$= \left[\frac{c_1 r_1}{c_1+r_1} + \frac{c_2 r_2}{c_2+r_2} + \frac{c_3 r_3}{c_3+r_3} + \frac{c_4 r_4}{c_4+r_4}\right]\frac{q}{8} + (s_1+s_2+s_3+s_4)\frac{R}{4q} + Sq + \beta\theta q$$

$$= F(q) \qquad (13)$$

Computation of q_d * at which F(q) **is minimum:**

F(q) is minimum when $\frac{\partial F(q)}{\partial q} = 0$ and where $\frac{\partial^2 F(q)}{\partial q^2} > 0$

Now, $\frac{\partial F(q)}{\partial q} = 0$, gives the economic order quantity as:

$$q_d^* = \sqrt{\frac{2(s_1+s_2+s_3+s_4)R}{\frac{c_1 r_1}{c_1+r_1} + \frac{c_2 r_2}{c_2+r_2} + \frac{c_3 r_3}{c_3+r_3} + \frac{c_4 r_4}{c_4+r_4} + 8S + 8\beta\theta}} \qquad (14)$$

Also, at **q** = q_d^* , we have $\frac{\partial^2 F(q)}{\partial q^2} > 0$. This shows that F(q) is minimum at **q** = q_d^*

And from equation (13), we get:

$$F(q)^* = \left[\frac{c_1 r_1}{c_1+r_1} + \frac{c_2 r_2}{c_2+r_2} + \frac{c_3 r_3}{c_3+r_3} + \frac{c_4 r_4}{c_4+r_4}\right]\frac{q}{8} + (s_1+s_2+s_3+s_4)\frac{R}{4q} + Sq + \beta\theta q \qquad (15)$$

NUMERICAL EXAMPLE

Numerical Example in Crisp Sense

The annual demand of an item is Rs. 800 unit / year. Annual inventory holding cost is Rs.16 per unit, set up cost is Rs.30 per unit and shortage cost is Rs. 12 unit / year. If there is 10 % defective items then the duplicate cost for the defective items is Rs. 3 / unit and the screening cost is Rs. 5 / unit. Economic order quantity and total annual cost are determined.

Sol:

R = 800 units / year
c = Rs. 16/ unit / year
r = Rs. 12 / unit / year
s = Rs. 30 / unit / year
θ = 10 %
S = Rs. 3 / unit
β = Rs. 5 / unit

1) Economic order quantity :

$$q* = \sqrt{\frac{2Rs(c+r)}{cr + 2S(c+r) + 2\beta\theta(c+r)}}$$

= 58.85 units

2) Total annual cost:

$$TC* = \sqrt{2Rs\left[\frac{cr}{c+r} + 2S + 2\beta\theta\right]}$$

= Rs. 813.87

Numerical Example in Fuzzy Sense

Let

R = 800 units / year

$\tilde{c}$ = Rs. (12, 15,17,20) / unit / year

$\tilde{r}$ = Rs. (8, 11, 13, 16) / unit / year

$\tilde{s}$ = Rs. (26, 29, 31, 34) / unit / year

θ = 10 %

S = Rs. 3 / unit

β = Rs. 5 / unit

1) Economic order quantity :

$$q_d^* = \sqrt{\frac{2(s_1+s_2+s_3+s_4)R}{\frac{c_1r_1}{c_1+r_1}+\frac{c_2r_2}{c_2+r_2}+\frac{c_3r_3}{c_3+r_3}+\frac{c_4r_4}{c_4+r_4}+8S+8\beta\theta}}$$

= 58.86 units

2) Total annual cost:

$$F(q)^* = \left[\frac{c_1r_1}{c_1+r_1}+\frac{c_2r_2}{c_2+r_2}+\frac{c_3r_3}{c_3+r_3}+\frac{c_4r_4}{c_4+r_4}\right]\frac{q}{8}+ (s_1+s_2+s_3+s_4)\frac{R}{4q}+Sq+\beta\theta q$$

= Rs. 813.88

Table 1: Sensitivity Analysis

S. No	Demand (R)	For $\tilde{c}$ = Rs. (12,15,17,20) $\tilde{r}$ = Rs. (8,11,13,16) $\tilde{s}$ = Rs.(26,29,31, 34)		For $\tilde{c}$ = Rs.(12, 14,18,20) $\tilde{r}$ =Rs. (8, 10, 14, 16) $\tilde{s}$ = Rs.(26, 28, 32, 34)	
		q_d^*	$F(q)^*$	q_d^*	$F(q)^*$
1.	**700**	**55.06**	**761.31**	**55.06**	**761.31**
2.	**750**	**57**	**788.04**	**57**	**788.04**
3.	**800**	**58.86**	**813.88**	**58.86**	**813.88**
4.	**850**	**60.68**	**838.93**	**60.68**	**838.93**
5.	**900**	**62.44**	**863.26**	**62.44**	**863.26**

NUMERICAL EXAMPLE

Numerical Example in Crisp Sense

A company uses 20,000 units of raw material which costs 50 rupees per unit. Placing each order costs rupees 30, carrying cost 20% per year of the inventory and shortage costs rupees 15 per year. If there is 10 % defective items then the duplicate cost for the defective items is Rs. 5 / unit and the screening cost is Rs. 3 / unit. Economic order quantity and total annual cost are determined.

Sol:

R = 20000 units / year

c = Rs. 50 X 20 % = Rs. 10 / unit / year

r = Rs. 15 / unit / year

s = Rs. 30 / unit / year

θ = 10 %

S = Rs. 5 / unit

β = Rs. 3 / unit

1) Economic order quantity :

$$q* = \sqrt{\frac{2Rs(c+r)}{cr+2S(c+r)+2\beta\theta(c+r)}}$$

= 268.87 units

2) Total annual cost:

$$TC* = \sqrt{2Rs\left[\frac{cr}{c+r}+2S+2\beta\theta\right]}$$

= Rs. 4463.18

Numerical Example in Fuzzy Sense

Let

R = 20,000 units / year

$\tilde{c}$ = Rs. (6, 9,11,14) / unit / year

$\tilde{r}$ = Rs. (11, 14, 16, 19) / unit / year

$\tilde{s}$ = Rs. (26, 29, 31, 34) / unit / year

θ = 10 %

S = Rs. 5 / unit

β = Rs. 3 / unit

1) Economic order quantity :

$$q_d^* = \sqrt{\frac{2(s_1+s_2+s_3+s_4)R}{\frac{c_1r_1}{c_1+r_1}+\frac{c_2r_2}{c_2+r_2}+\frac{c_3r_3}{c_3+r_3}+\frac{c_4r_4}{c_4+r_4}+8S+8\beta\theta}}$$

= 268.90 units

2) Total annual cost:

$$F(q)^* = \left[\frac{c_1r_1}{c_1+r_1}+\frac{c_2r_2}{c_2+r_2}+\frac{c_3r_3}{c_3+r_3}+\frac{c_4r_4}{c_4+r_4}\right]\frac{q}{8}+ (s_1+s_2+s_3+s_4)\frac{R}{4q}+Sq+\beta\theta q$$

= Rs. 4463.18

Table 2: Sensitivity Analysis

S. No	Demand (R)	For $\widetilde{c}$ =Rs.(6,9,11,14) $\widetilde{r}$ =Rs.(11, 14, 16, 19) $\widetilde{s}$ =Rs.(26, 29, 31, 34)		For $\widetilde{c}$ = Rs. (6, 8,12,14) $\widetilde{r}$ =Rs.(11, 13, 17, 19) $\widetilde{s}$ =Rs.(26, 28, 32, 34)	
		q_d^*	$F(q)^*$	q_d^*	$F(q)^*$
1.	18000	255.18	4231.86	255.18	4231.86
2.	19000	262.18	4348.20	262.18	4348.20
3.	20000	268.90	4461.16	268.90	4461.16
4.	21000	275.63	4571.33	275.63	4571.33
5.	22000	282.11	4678.90	282.11	4678.90

CONCLUSION

In this paper, we develop a economic order quantity and total annual inventory cost in the crisp sense as well as fuzzy sense. Carrying cost, set up cost and shortage cost are taken as a trapezoidal fuzzy numbers. Here we acquire the defective items in terms of percentage; screening and reworking costs are taken as constant. This model is solved analytically through JIT by minimizing the total inventory cost. Finally, the proposed model has been verified by the two numerical example along with the sensitivity analysis. In the future study, we apply the fuzzy concept for all provisions in this anticipated model.

REFERENCES

Chan W., 1996. Backorder fuzzy inventory model under function principle, Information Science, **95**(1-2):71-79.

Dutta D. and Kumar P., 2012. Fuzzy inventory model without shortage using trapezoidal fuzzy number with sensitivity analysis, IOSR Journal of Mathematics, **4**(3):32-37, ISSN: 2278-5728.

Hadley G. and Whitin T.M., 1963. Analysis of inventory systems, Prentice-Hall, Englewood clipps, NJ, 1963.

Harris F., 1915. Operations and cost, AW Shaw Co. Chicago.

Hsieh C.H., 2002. Optimization of Fuzzy Production Inventory Models, Information Sciences, **146**(1-4):29-40.

Jain R., 1976. Decision making in the presence of fuzzy variables, IIIE Transactions on systems, Man and Cybernetics, **17**:698- 703.

Kacpryzk J. and Staniewski P., 1982. Long-term inventory policy-making through fuzzy-decision making models, Fuzzy Sets and Systems, **8**:117-132.

Kao C.K. and Hsu W.K., 2002. A single-period inventory model with fuzzy demand, Computers and Mathematics with Applications, **43**:841-848.

Park K.S., 1987. Fuzzy Set Theoretical Interpretation of economic order quantity, IEEE Trans. Systems Man. Cybernet SMC, **17**:1082-1084.

De P.K. and Rawat A., 2011. A fuzzy inventory model without shortages using triangular fuzzy number, Fuzzy Information & Engineering, **1**:59-68, 2013.DOI: 10.3844/ajassp.2013.

Slack N., Chambers S., Harland C., Harrison A. and Johnston R., 1995. Operations Management, Pitman Publishing, London.

Syed J.K. and Aziz L.A., 2007. Fuzzy inventory model without shortages using signed distance method, Applied Mathematics &Information Sciences, **1**(2):203-209.

Urgeletti Tinarelli G., 1983. Inventory control models and problems, European Journal of Operational Research, **14**:1-12.

Vujosevic M., Petrovic D. and Petrovic R., 1996. EOQ Formula when Inventory Cost is Fuzzy, International Journal of Production Economics, **45**:499-504.

Wilson R., 1934. A scientific routine for stock control. Harvard Business Review, **13**:116–128.

Yao J.S. and Chiang J., 2003. Inventory without back order with fuzzy total cost and fuzzy storing cost defuzzified by centroid and signed distance, European Journal of Operational Research, **148**:401-409.

Yao J.S. and Lee H.M., 1999. Fuzzy Inventory with or without backorder for fuzzy order quantity with trapezoidal fuzzy number, Fuzzy sets and systems, **105**:311- 337.

Yao J.S. and Lee H.M., 1999. Economic order quantity in fuzzy sense for inventory without backorder model, Fuzzy Sets and Systems, **105**:13-31.

Zadeh L.A., 1965. Fuzzy sets, Information Control, **8**:338-353.

Zadeh L.A. and Bellman R.E., 1970. Decision Making in a Fuzzy Environment, Management Science, **17**:140-164.

Zimmerman H.J., 1983. Using fuzzy sets in operational Research, European Journal of Operational Research, **13**:201-206.