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ANTI FUZZY d – IDEALS IN d – ALGEBRA

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ABSTRACT

In this paper, we introduce the notion of anti fuzzy d–Ideals of d-algebras and prove some results. Homomorphism functions are satisfied while applying anti fuzzy d – ideal concepts and Cartesian Product is also applied.

KEYWORDS: d – algebra, d – Ideal, Fuzzy d – Ideal, Anti Fuzzy d – Ideal, Anti Fuzzy d – algebra Homomorphism and Cartesian Product.

After the introduction of fuzzy subsets by Zadeh L.A., 1965, several researchers explored on the generalization of the notion of fuzzy subset. Neggers J. and Kim H.S., 1999, introduced the class of d-algebras which is another generalization of BCK-algebras, and investigated relations between d-algebras and BCK-algebras. Akram M. and Dar H.H., 2005, fuzzified d – algebra. Mostafa et. al., 2012, introduced Anti – Fuzzy KU ideals of Ku – algebras. Priya T. and Ramachandran T., 2014, introducted Anti Fuzzy PS – Ideals of PS – algebras. Modifying their idea, in this paper, we apply the idea to d-Algebra. We introduce the notion of Anti Fuzzy d-ideals of d-Algebras, Homomorphism and Cartesian Product of Anti Fuzzy d - ideals, and prove some results on these.

PRELIMINARIES

In this section we give some basic definitions and preliminaries of d-algebras and introduce Anti Fuzzy d – algebra and Anti Fuzzy d – ideals .

Definition 2.1: (Neggers J. and Kim H.S., 1999)

A non – empty set X with a constant 0 and a binary operation * is called a d-algebra if it satisfies the following axioms:

(i) $x * x = 0$

(ii) $0 * x = x$

(iii) $x * y = 0 \text{ and } y * x = 0 \Rightarrow x = y, \text{for all } x, y \in X$

Definition 2.2: (Neggers J. and Kim H.S., 1999)

Let X be a d – algebra and I be a subset of X, then I is called d – ideal of X if it satisfies following conditions:

(i) $0 \in I$

(ii) $x * y \in I \text{ and } y \in I$ implies $x \in I$ i.e., $I \times I \subseteq I$

Definition 2.3: (Zadeh L.A., 1965)

Let X be a non-empty set. A fuzzy subset β of the set X is a mapping

$\beta : X \to [0,1]$

Definition 2.4: (Akram M. and Dar H.H., 2005)

A fuzzy set β in a d – algebra X is called a fuzzy d – sub algebra of X if $\beta(x * y) \geq \beta(x) \wedge \beta(y) \; \forall x, y \in X$.

Definition 2.5: (Akram M. and Dar H.H., 2005)

A fuzzy set β in X is called fuzzy d – ideal of X if it satisfies the following inequalities:

(i) $\beta(0) \geq \beta(x)$

(ii) $\beta(x) \geq \beta(x * y) \wedge \beta(y)$

(iii) $\beta(x * y) \geq \beta(x) \wedge \beta(y), \forall x, y \in X$

Let $X := \{0,1,2\}$ be a d – algebra with the following table:

*	0	1	2
0	0	0	0
1	1	0	1
2	2	2	0

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We define β by $\beta(0) = 0.7, \beta(1) = 0.5, \beta(2) = 0.2$, then β is a d – ideal of X.

Definition 2.6: (Priya T. and Ramachandran T., 2014)

Let X be a d – algebra. A fuzzy set β in X is called an anti fuzzy d – ideal of X if it satisfies the following conditions:

(i) $\beta(0) \leq \beta(x)$

(ii) $\beta(x) \leq \beta(x * y) \wedge \beta(y)$

(iii) $\beta(x * y) \leq \beta(x) \wedge \beta(y), \forall x, y \in X$

Let $X := \{0, a, b, c\}$ be a d – algebra with the following table:

*	0	A	b	c
0	0	0	0	0
A	a	0	0	a
B	b	B	0	0
C	c	c	c	0

We define β by $\beta(0) = 0.1, \beta(a) = 0.2, \beta(b) = 0.4, \beta(c) = 0.7$ then β is an anti fuzzy d – ideal of X.

Definition 2.7: (Priya T. and Ramachandran T., 2014)

A fuzzy set β is a d – algebra X is called an anti fuzzy d – sub algebra of X if $\beta(x * y) \leq \beta(x) \vee \beta(y), \forall\ x, y \in X$.

HOMOMORPHISM OF ANTI FUZZY d – IDEALS AND ANTI FUZZY d – SUB ALGEBRA

Definition 3.1:

Let $(X, *, 0)$ and $(Y, *, 0')$ be d – algebras. A mapping $f: X \rightarrow Y$ is said to be a homomorphism if $f(x * y) = f(x) * f(y)\ \forall x, y \in X$.

Note 3.2:

If $f: X \rightarrow Y$ is homomorphism of d – algebra then $f(0) = 0'$.

Definition 3.3:

Let $f: X \rightarrow X$ be an endomorphism and β be a fuzzy set in X. We define a new fuzzy set in X by $\beta_f(x) = \beta(f(x)) \forall x$ in X.

Theorem 3.4:

Let f be an endomorphism of d – algebra of X. If β is an anti fuzzy d – ideal of X then so is β_f.

Proof

Let β be an anti fuzzy d – ideal of X.

Now,
$\beta_f = \beta(f(0)) \leq \beta(f(x)) = \beta_f(x)\ \forall x \in X$

Let $x, y \in X$

$\beta_f(x) = \beta(f(x)) \leq \beta(f(x) * f(y)) \vee \beta(f(y))$

$= \beta(f(x * y)) \vee \beta(f(y))$

$= \beta_f(x * y) \vee \beta_f(y)$

Then,

$\beta_f(x * y) = \beta(f(x * y))$

$\leq \beta(f(x)) \vee \beta(f(y))$

$= \beta_f(x) \vee \beta_f(y)$

Hence, β_f is an anti fuzzy d – ideal of X.

Theorem 3.5:

Let $f: X \rightarrow X$ be an endomorphism on a d – algebra X. If β be an anti fuzzy d – sub algebra of X, then β_f is an anti fuzzy d – sub algebra of X.

Proof

Let β be an anti fuzzy d – sub algebra of X. Let $x, y \in X$.

Now,

$\beta_f(x * y) = \beta(f(x * y))$

$= \beta(f(x) * f(y))$

$\leq \beta(f(x)) \vee \beta(f(y))$

$= \beta_f(x) \vee \beta_f(y)$

$\Rightarrow$ β_f is an anti fuzzy d – sub algebra of X.

Hence, the proof.

CARTESIAN PRODUCT OF ANTI FUZZY d – IDEALS OF d – ALGEBRA

Definition 4.1:

Let β and γ be the anti fuzzy sets in X. The Cartesian product $\beta \times \gamma : X \times X \rightarrow [0,1]$ is defined by $(\beta \times \gamma)(x,y) = \beta(x) \vee \gamma(y) \forall x,y \in X$.

Theorem 4.2:

If β and γ are anti fuzzy d – ideals of a d algebra X. The $\beta \times \gamma$ is anti fuzzy d – ideal of $X \times X$.

Proof

For any $(x,y) \in X \times X$, we have

$(\beta \times \gamma)(0,0) = \beta(0) \vee \gamma(0) \leq \beta(x) \vee \gamma(y)$

$= (\beta \times \gamma)(x,y)$

Let (x_1,x_2) and $(y_1,y_2) \in X \times X$. Then

$(\beta \times \gamma)(x_1,x_2) = \beta(x_1) \vee \gamma(x_2)$

$\leq (\beta(x_1 * y_1) \vee \beta(y_1)) \vee (\gamma(x_2 * y_2) \vee \gamma(y_2))$

$= (\beta(x_1 * y_1) \vee \gamma(x_2 * y_2)) \vee (\beta(y_1) \vee \gamma(y_2))$

$= (\beta \times \gamma)((x_1 * y_1),(x_2 * y_2)) \vee (\beta \times \gamma)(y_1,y_2)$

And,

$(\beta \times \gamma)((x_1,x_2) * (y_1,y_2)) = (\beta \times \gamma)(x_1 * y_1, x_2 * y_2)$

$= \beta(x_1 * y_1) \vee \gamma(x_2 * y_2)$

$\leq (\beta(x_1) \vee \beta(y_1)) \vee (\gamma(x_2) \vee \gamma(y_2))$

$= (\beta(x_1) \vee \gamma(x_2)) \vee (\beta(y_1) \vee \gamma(y_2))$

$= ((\beta \times \gamma)(x_1,x_2)) \vee (\beta \times \gamma)(y_1,y_2))$

Hence, $\beta \times \gamma$ is an anti fuzzy d – ideal of X.

Theorem 4.3:

Let β and γ be anti fuzzy sets in a d – algebra X such that $\beta \times \gamma$ is an anti fuzzy d – ideal of $X \times X$. Then

(i) either $\beta(0) \leq \beta(x)$ or $\gamma(0) \leq \gamma(x)$ $\forall x \in X$

(ii) if $\beta(0) \leq \beta(x) \forall x \in X$, then either $\gamma(0) \leq \beta(x)$ or $\gamma(0) \leq \gamma(x)$

(iii) if $\gamma(0) \leq \gamma(x) \forall x \in X$, then either $\beta(0) \leq \beta(x)$ or $\beta(0) \leq \gamma(x)$

Proof

Let $\beta \times \gamma$ be an anti fuzzy d – ideal of $X \times X$.

(i) Suppose that $\beta(0) > \beta(x)$ and $\gamma(0) > \gamma(x)$ for some $x, y \in X$.

Then

$(\beta \times \gamma)(x,y) = \beta(x) \vee \gamma(y) < \beta(0) \vee \gamma(0)$

$= (\beta \times \gamma)(0,0)$

$\Rightarrow (\beta \times \gamma)(x,y) < (\beta \times \gamma)(0,0)$

which is a contradiction. Hence (i) is proved.

(ii) Assume that there exists $x, y \in X$ such that $\gamma(0) > \beta(x)$ and $\gamma(0) > \gamma(y)$.

Then,

$(\beta \times \gamma)(x,y) = \beta(x) \vee \gamma(y) < \gamma(0)$

$= (\beta \times \gamma)(0,0)$

$\Rightarrow (\beta \times \gamma)(x,y) < (\beta \times \gamma)(0,0)$

which is a contradiction. Hence (ii) is proved.

Similarly, we can prove if $\gamma(0) \leq \gamma(x) \forall x \in X$, then either $\beta(0) \leq \beta(x)$ or $\beta(0) \leq \gamma(x)$.

Hence Proved.

Theorem 4.4:

If $\beta \times \gamma$ is an anti fuzzy d – ideal of $X \times X$, then β or γ is an anti fuzzy d – ideal of X.

Proof

Let $\beta \times \gamma$ be an anti fuzzy d – ideal of $X \times X$.

First we prove γ is an anti fuzzy d – ideal of X.

Since, by $4.3\ (i)$ either either $\beta(0) \leq \beta(x)$ or $\gamma(0) \leq \gamma(x)\ \forall x \in X$.

Assume that $\gamma(0) \leq \gamma(x)\ \forall x \in X$.

It follows from $4.3(iii)$ that either $\beta(0) \leq \beta(x)$ or $\beta(0) \leq \gamma(x)$.

If $\beta(0) \leq \gamma(x)$, for any $x \in X$, then

$$(\beta \times \gamma)(0,x) = \beta(0) \vee \gamma(x) = \gamma(x)$$

Since $\beta \times \gamma$ is an anti fuzzy d – ideal of $X \times X$, therefore,

$$(\beta \times \gamma)(x_1,x_2) \leq (\beta \times \gamma)\big((x_1,x_2) * (y_1,y_2)\big) \vee \big((\beta \times \gamma)(y_1,y_2)\big)$$

and

$$(\beta \times \gamma)\big((x_1,x_2) * (y_1,y_2)\big) \leq \big((\beta \times \gamma)(x_1,x_2)\big) \vee \big((\beta \times \gamma)(y_1,y_2)\big)$$

$$\Rightarrow (\beta \times \gamma)(x_1,x_2) \leq (\beta \times \gamma)(x_1 * y_1, x_2 * y_2) \vee \big((\beta \times \gamma)(y_1,y_2)\big)$$

and

$$(\beta \times \gamma)(x_1 * y_1, x_2 * y_2) \leq \big((\beta \times \gamma)(x_1,x_2)\big) \vee \big((\beta \times \gamma)(y_1,y_2)\big)$$

Putting $x_1 = y_1 = 0$, we have

$$(\beta \times \gamma)(0,x_2) \leq (\beta \times \gamma)(0, x_2 * y_2) \vee \big((\beta \times \gamma)(0,y_2)\big)$$

and

$$(\beta \times \gamma)(0,x_2 * y_2) \leq \big((\beta \times \gamma)(0,x_2)\big) \vee \big((\beta \times \gamma)(0,y_2)\big)$$

$$\Rightarrow \gamma(x_2) \leq \gamma(x_2 * y_2) \vee \gamma(y_2)$$

$$\gamma(x_2 * y_2) \leq \gamma(x_2) \vee \gamma(y_2)$$

This proves that γ is an anti fuzzy d – ideal of X.

The second part is similar. This completes the proof.

CONCLUSION

Hence we have discussed the Anti Fuzzy d-ideals on d- algebras. Which adds an another dimension to the defined Anti Fuzzy d- algebras. This concept can further be generalized to Normalization of fuzzy d- ideals in d- algebras, n-fold fuzzy d- ideals on n-fold fuzzy d-algebras for new results in our future work.

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