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SOLUTION OF SYSTEM OF DIFFERENTIAL EQUATIONS USING DIFFERENTIAL TRANSFORM METHOD

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ABSTRACT

Aim of the paper is to obtain solution of system of differential equations applied in science and engineering using Differential Transform Method (DTM). The solutions obtained by DTM are compared with solution obtained by other methods. It is observed that DTM method converges fast and save lot of time and avoid computational volume.

KEYWORDS: Differential Transform Method, Homotopy Analysis Method, Stiff Systems, System of Differential Equations

Differential equations play an important part in Applied Sciences and Engineering. There has been continuous research going on in developing new techniques for solving physical problems involving differential equations. The purpose of new techniques is to obtain analytical or numerical solutions of differential equations with higher accuracy and fast convergence that save lot of time and avoid computational complexity.

In stiff differential equations of type $z' = g(t,z)$ the solution contains terms that decreases to zero as t increases. Stiff differential equations are used in different areas of science and technology. To obtain the solution of such differential equations has always been a challenging task before mathematicians and engineers.

Zhou [7] used idea of Differential Transform Method (DTM) to obtain solution of linear and nonlinear problems in circuit theory. Hassan [6] obtained analytical solution for different systems of differential equations using Differential transform method. Arikoglu and Ozkol [10] applied DTM to solve the integro differential equation. Kanth and Aruna [11] solved linear and nonlinear Klein Gordon equation using DTM.

Aminikhah [14] applied Laplace transform and Homotopy perturbation method to solve stiff systems of ordinary differential equations. Yüzbaşı and Şahin [12] obtained numerical solution of parabolic convection diffusion problems using Bessel collocation method. Zhaiet. al [13] investigated poission equation using difference schemes. Addulkawi [9] extended the DTM to solve Cauchy type singular integral equations over a finite interval. Taghavi et. al[15] used DTM for solving nonlinear reaction diffusion convection equation. They used wave variable to convert partial differential equation into ordinary differential equation and then used DTM to obtain solution. Mohmoud and Gubara [16] used reduced differential transform method to find approximate analytical solution of the problem in the series form. Further Singh and Mahendra [17] investigated analytical solution of an initial value system of time dependent linear and nonlinear partial differential equation using reduced differential transform method. Mohand et.al [18] obtained analytical solution of some nonlinear differential equations using new integral transform with DTM

Aim of the paper is to apply Differential Transform Method (DTM) to stiff differential equation, Genesio system of equation and wave equation and compared the solution obtained by other methods

ANALYSIS OF THE METHOD

Let $h(x,t)$ be continuously differential function of x and t defined in prescribed domain then

$$H_k(x) = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} h(x,t)\right]_{t=0} \qquad (1)$$

where $H_k(x)$ is the differential transformed function.

The inverse differential transform of $H_k(x)$ is given by

$$h(x,t) = \sum_{k=0}^{\infty} H_k(x) t^k \qquad (2)$$

using equations (1) and (2) together

$$h(x,t) = \sum_{k=0}^{\infty} t^k \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} h(x,t)\right]_{t=0} \qquad (3)$$

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APPLICATIONS

Example 1

Consider the stiff system of equations [1]
$$\frac{dx_1}{dt} = k^{'}(-1-\varepsilon)x_1 + k^{'}(1-\varepsilon)x_2 \quad (4)$$

$$\frac{dx_2}{dt} = k^{'}(1-\varepsilon) + (-1-\varepsilon)k^{'}x_2 \quad (5)$$

and the initial conditions are

$$x_1(0) = 1,\ x_2(0) = 3 \quad (6)$$

where $k^{'}$ and ε are constants

Taking the differential transform of both sides of equations (4) and (5), we get

$$(k+1)X_1(k+1) = k^{'}(-1-\varepsilon)X_1(k) + k^{'}(1-\varepsilon)X_2(k) \quad (7)$$

$$(k+1)X_2(k+1) = k^{'}(1-\varepsilon)X_1(k) + k^{'}(-1-\varepsilon)X_2(k) \quad (8)$$

The various components of series solution are

$X_1(0) = 1$, $X_1(1) = 2k^{'} - 4k^{'}\varepsilon$, $X_1(2) = -2k^{'2} + 4\varepsilon^2 k^{'2}$ and so on. (9)

$X_2(0) = 3$, $X_2(1) = -2k^{'} - 4k^{'}\varepsilon$, $X_2(2) = 2k^{'2} + 4\varepsilon^2 k^{'2}$ and so on. (10)

Then the solution is given by

$$x_1 = \sum_{k=0} X_1(k)t^k \quad (11)$$

$$= -e^{-2k^{'}t} + 2e^{-2\varepsilon tk^{'}} \quad (12)$$

similarly

$$x_2 = \sum_{k=0} X_2(k)t^k = e^{-2kt} + e^{-2k\varepsilon t} \quad (13)$$

which are the same solution obtained by Bataineh et .al [5] using Homotopy Analysis Method For $k = 50$ and $\varepsilon = 0.01$ then Solutions are

$$x_1(t) = -e^{-100t} + 2e^{-t} \quad (14)$$
$$x_2(t) = e^{-100t} + 2e^{-t} \quad (15)$$

which are the same solutions obtained by Bataineh et .al [5] using Homotopy Analysis Method

Example 2

Consider the following system of stiff nonlinear equations [2]

$$\frac{dx_1}{dt} = -1002x_1 + 1000x_2^2 \quad (16)$$

$$\frac{dx_2}{dt} = x_1 - x_2 - x_2^2 \quad (17)$$

with the conditions

$$x_1(0) = 1,\ x_2(0) = 1 \quad (18)$$

Taking the differential transform of equations (16) and (17), we get

$$(k+1)X_1(k+1) = -1002X_1(k) + 1000\sum_{k_1}^{k} X_2^{k_1} X_2^{k-k_1} \quad (19)$$

$$(k+1)X_2(k+1) = X_1(k) - X_2(k) - \sum_{k_1}^{k} X_2^{k_1} X_2^{k-k_1} \quad (20)$$

The various components of series are

$X_1(0) = 1$, $X_1(1) = -2$, $X_1(2) = 2$ and so on. (21)

$X_2(0) = 1$, $X_2(1) = -1$, $X_2(2) = \frac{1}{2}$ and so on. (22)

Then the solution is given by

$$x_1 = \sum_{k=0} X_1(k)t^k \quad (23)$$

$$= 1 - 2t + \frac{(2t)^2}{2!} - ..... \quad (24)$$

$$= e^{-2t} \quad (25)$$

similarly

$$x_2 = \sum_{k=0} X_2(k)t^k \quad (26)$$

$$= e^{-t} \quad (27)$$

which is the same solutions obtained by Bataineh et .al [5] using Homotopy Analysis Method

Example 3

Consider the following Genesio system of equations [3]

$$\frac{dx}{dt} = y \quad (28)$$

$$\frac{dy}{dt} = z \quad (29)$$

$$\frac{dz}{dt} = -cx - by - az + x^2 \quad (30)$$

with the initial condition

$x(0) = 0.2$, $y(0) = -0.3$, $z(0) = 0.1$ (31)

Taking the DTM of given equations (28), (29) and (30), we get

$$(k+1)X(k) = Y(k) \quad (32)$$

$$(k+1)Y(k+1) = Z(k) \quad (33)$$

$$(k+1)Z(k+1) = -cX(k) - bY(k) - aZ(k) + \sum_{k_1=0}^{k} X_{k_1} X_{k-k_1} \quad (34)$$

taking $a = 1.2, b = 2.92, c = 6$

The various component of series are

$X(0) = 0.2$, $X(1) = -0.3$, $X(2) = 0.05$ and so on (35)

$Y(0) = -0.3$, $Y(1) = 0.1$, $Y(2) = -0.202$ and so on (36)

$Z(0) = 0.1$, $Z(1) = -0.202$, $Z(2) = 0.9364$ and so on.(37)

Then the solution is given by

$$x = \sum_{k=0} X(k)t^k \quad (38)$$

$=0.2-0.3\ t + 0.05\ t^2 + \ldots\ldots\ldots\ldots\ldots\ldots$ (39)

$$y = \sum_{k=0} Y(k)t^k \quad (40)$$

$$z = \sum_{k=0} Z(k)t^k$$

$=-0.3+0.1\ t - 0.202\ t^2 + \ldots\ldots\ldots\ldots\ldots\ldots$ (41)

(42)

$=0.1-0.202\ t + 0.9364\ t^2 + \ldots\ldots\ldots\ldots\ ..$ (43)

which is the same solution obtained by Bataineh et .al [5] using Homotopy Analysis Method

Example 4

Consider the following wave equation [8]

$u_{tt} = u_{xx}, -\infty < x < \infty, t > 0$ (44)

initial condition $u(x,0) = \sin x$ (45)

and boundary condition $u_t(x,0) = \cos x$ (46)

Taking the DTM of given equations (44) and (46), we get

$(k+1)U_k = \cos x$ and $(k+1)(k+2)U_{k+2} = \frac{\partial^2}{\partial x^2} U_k$ (47)

The various component of series solution are

$U_0 = \sin x$, $U_1 = \cos x$, $U_2 = \frac{-1}{2!} \sin x$, $U_3 = \frac{-\cos x}{3!}$ and so on. (48)

Then the solution is given by

$$u(x,t) = \sum_{k-0} U(k)t^k \quad (49)$$

$$= \sin(x+t) \quad (50)$$

which is the same solution obtained by Chun et.al [4] using Homotopy Perturbation method

CONCLUSION

Differential transform method is applied to solve Stiff differential equations. The method does not require linearization, discretization or restrictive assumptions. It is evident that DTM needs small size of computation contrary to other numerical methods and its speed of the convergence is very fast and reliable.

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