

TIME SERIES FORECASTING MODEL FOR IRANIAN GOLD PRICE

MEHDI BASIKHASTE^{a1}, MOHAMAD A. MOVAFAGHPUR^b AND MARYAM SAADATIFARD^c^aDepartment of Mathematics, Dezful Branch, Islamic Azad University, Dezful, Iran^bJundi Shapur University of Technology, Dezful, Iran^cDepartment of Accounting, Dezful Branch, Islamic Azad University, Dezful, Iran

ABSTRACT

Gold demand in recent years due to rising prices and the economic crisis due to sanctions on Iran has increased. In this paper we have used AR model, AR-IGARCH model, SETAR and STAR models for forecasting and these methods are applied for modeling a monthly log return time series of gold price from August 2007 to November 2013 (price in Iranian Rial against 1 gram of gold). The result reviews that the time series is nonlinear and SETAR (2,1,3) model yields the best result.

KEYWORDS: Log return time series; ARCH/GARCH model; TAR model

Financial time series analysis is the fundamental tool of study in asset valuation over time. We know that one of the reasons for the change in the price of gold is the external factors such as economic policy, environmental, political and social issues. In this hypothesis the external effects are modeled as noise, and the phenomena one considered as accidental. When the price of a good is increased, usually a self-regulating force decreases the prices, and vice versa when a price decreased; this force caused the prices to go high. This feedback mechanism could be linear or nonlinear[3]. A time series is stationary if its properties are statistically invariant over time and can be modeled by various parametric methods. ARMA models are used to model the conditional expectation of a process given the past, but in an ARMA model the conditional variance given the past is constant. Generalized Auto regressive Conditional Heteroscedastic (GARCH) model consider the moments of a time series as invariant and is one approach to modeling time series with heteroscedastic errors. Threshold Autoregressive (TAR) models are commonly referred to as piecewise linear models or regime-switching models. The Self-Exciting Threshold Autoregressive (SETAR) model, first introduced by Tong[10], is a special case of the TAR model. Here, the movements between the regimes are controlled or governed by a variable called threshold just as in the TAR model with the difference that the threshold of a SETAR model is Self-Exciting. This means that, unlike the TAR model, where the threshold is assumed to be an exogenous variable, the threshold variable of a SETAR model is a certain lagged value of the series itself, an endogenous variable. Smooth Transition Autoregressive (STAR) models are typically applied to time series data as

an extension of autoregressive models, in order to allow for higher degree of flexibility in model parameters through a smooth transition. Assuming that behavior of the series changes depending on the value of the transition variable. The transition might depend on the past values of the data series (similar to the SETAR models), or exogenous variables.

Many financial studies are based on returns, rather than prices of assets. Campbell, Lo, and MacKinlay (1997) give two main reasons for using returns. First, for average in vestors, return of an asset is a complete and scale-free summary of the investment to opportunity. Second, return series are easier to handle than price series because the former have more attractive statistical properties.

Let P_t be the price of an asset at time index t . We introduce the definition of return that is used throughout the paper. The natural logarithm of the simple gross return of an asset is called the continuously compounded return or log return [1]:

$$r_t = \ln(1 + R_t) = \ln \frac{P_t}{P_{t-1}}$$

We organize the paper as below. Section 2 deals with ARCH/GARCH method and section 3 deals with Threshold Auto Regressive (TAR) model. In section 4 the analysis results of the monthly log return of gold price (price in Iranian Rial against 1 gram of gold) time series [August 2007 - November 2013] using these methods is presented and in section 5 the comparison of applied methods is discussed. Final conclusions are given in section 6.

ARCH/GARCH MODELS

Volatility is an important factor in financial applications and it means the conditional standard deviation of the underlying asset return. An Autoregressive Conditional Heteroscedastic model considers the variance of the current error term or innovation to be a function of the actual sizes of the previous time periods error terms. Often the variance is related to the squares of the previous innovations.

The univariate volatility models include the autoregressive conditional heteroscedastic (ARCH) model of Engle (1982) and the generalized ARCH(GARCH) model of Bollerslev (1986).

ARCH Model

The first model that provides a systematic framework for volatility modeling is the ARCH model of Engle (1982). The basic idea of ARCH models is that

- (a) The shock a_t of an asset return is serially uncorrelated, but dependent, and
- (b) The dependence of a_t can be described by a simple quadratic function of its lagged values. Specifically, an ARCH(m) model assumes that

$$r_t = \mu_t + a_t, \quad a_t = \sigma_t \epsilon_t, \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \dots + \alpha_m a_{t-m}^2,$$

Where $\epsilon_t \sim \text{IID}(0,1)$, $\alpha_0 > 0$, and $\alpha_i \gg 0$ for $i > 0$. The equation of μ_t should be an econometric model (e.g., an ARMA model) for the return series to remove any linear dependence. The coefficients α_i must satisfy some regularity conditions to ensure that the unconditional variance of a_t is finite. In practice, ϵ_t is often assumed to follow the standard normal or a standardized Student-t or a generalized error distribution [1].

GARCH Model

Bollerslev (1986) proposes a useful extension known as the generalized ARCH (GARCH) model. For a log return series r_t , let $a_t = r_t - \mu_t$ be the innovation at time t . Then a_t follows a GARCH(m, s) model if

$$a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \alpha_0 + \sum_{i=1}^m \alpha_i a_{t-i}^2 + \sum_{j=1}^s \beta_j \sigma_{t-j}^2,$$

Where $\epsilon_t \sim \text{IID}(0,1)$, $\alpha_0 > 0$, $\alpha_i \gg 0$, $\beta_j \gg 0$, and $\sum_{i=1}^{\max(m,s)} (\alpha_i + \beta_j) < 1$. As before, ϵ_t is often assumed to

follow a standard normal or standardized Student-t distribution or generalized error distribution. The above equation reduces to a pure ARCH(m) model if $s = 0$. The α_i and β_j are referred to as ARCH and GARCH parameters, respectively [1].

The Integrated GARCH Model

If the AR polynomial of the GARCH representation has a unit root then we have an IGARCH model [1]. An IGARCH(1,1) model can be written as

$$r_t = \mu_t + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \alpha_0 + \beta_1 \sigma_{t-1}^2 + (1 - \beta_1) a_{t-1}^2,$$

Where $0 < \beta_1 < 1$.

THRESHOLD AUTOREGRESSIVE (TAR) MODELS

The Threshold Autoregressive (TAR) family proposed and explained by Tong (1983) is contained within the state-dependent (regime-switching) model family [6]. The simplest class of TAR models is the Self Exciting Threshold Autoregressive (SETAR) models. The two regimes SETAR model is given by the following formula:

$$r_t = \begin{cases} \phi_0^{(1)} + \phi_1^{(1)} r_{t-1} - \dots - \phi_{p_1}^{(1)} r_{t-p_1} + \sigma_1 \epsilon_t, & \text{if } r_{t-d} \leq \gamma \\ \phi_0^{(2)} + \phi_1^{(2)} r_{t-1} - \dots - \phi_{p_2}^{(2)} r_{t-p_2} + \sigma_2 \epsilon_t, & \text{if } r_{t-d} > \gamma \end{cases}$$

Where γ is the threshold and d is the delay parameter. $\{\epsilon_t\}$ is a Gaussian white noise sequence with mean zero and variance 1. TAR models are piecewise linear, p_1 is AR order of the lower regime and p_2 is AR order of the upper regime. And finally σ_1, σ_2 are respectively standardized deviations in the lower and upper regimes. Such a process makes the model nonlinear for at least two regimes, but remains locally linear (Tsay, 1989). A comparatively recent development is the Smooth Transition Autoregressive (STAR) model, developed by Terasvirta and Anderson (1992). STAR models, unlike standard TAR models, allow for a more gradual transition of the dependent variable between regimes. The regime indicator in these models is a continuous function rather than an on-off switch (Brooks, 2002). The 2- regime STAR model of order p is defined by

$$r_t = c_0 + \sum_{i=1}^p \phi_0^{(i)} r_{t-i} + F(r_{t-d})(c_1 + \sum_{i=1}^p \phi_1^{(i)} r_{t-i}) + a_t$$

Where d is the delay parameter, and $F(\cdot)$ is a smooth transition function. In practice, $F(\cdot)$ often assumes one of three forms—namely, logistic, exponential, or a cumulative distribution function.

The logistic STAR (LSTAR) is an extension of the standard STAR model with the following transition function:

$$F(r_{t-d}) = \left(1 + \exp\left(-\frac{r_{t-d} - th}{\gamma}\right)\right)^{-1}$$

Where r_{t-d} is the threshold variable, th is transition parameter and γ is standard deviation of the threshold variable.

MODEL ESTIMATION AND PREDICTION OF LOG RETURN TIME SERIES

In recent years gold demand due to rising prices and the economic crisis due to sanctions on Iran has increased. The gold price time series is one of the most important economic time series. This particular time series is involved socially because investors prefer to deposit in gold whenever a negative swing in the economy caused.

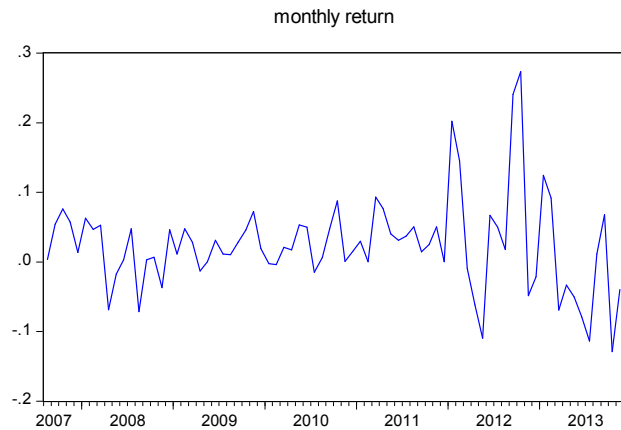


Figure 1: Time plot of monthly log returns

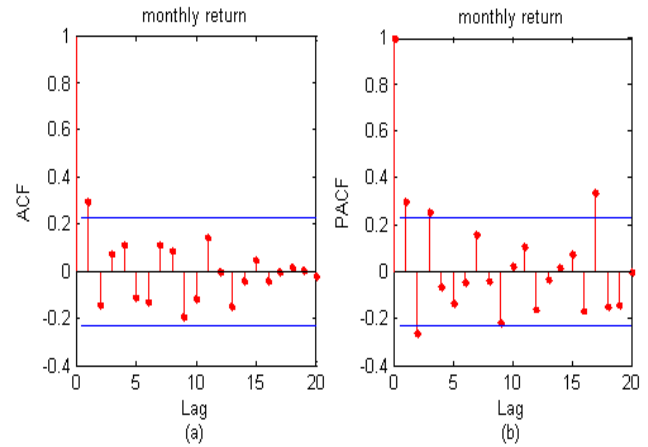


Figure 2: Sample ACF and PACF of monthly log

Also, the currency exchange rate is closely connected to the price of gold and the reserve of gold kept by the central bank of the government [11]. So this analysis is revealing in many different ways. Here we analyze the time series of monthly log return gold price from August 2007 to November 2013 using four different models. The plot of the time series is shown in figure 1. The auto correlation function of the time series is given in figure 2.

Estimation of ARMA Model to Log Returns

An AR(3) model is estimated for the monthly log returns of gold price. The fitted model is

$$r_t = 0.024 + 0.428r_{t-1} - 0.353r_{t-2} + 0.25r_{t-3} + a_t$$

$$\hat{\sigma}_a = 0.062, R^2 = 0.198, mse = 0.0038$$

The plot of the given time series and its prediction using the estimated AR(3) model is given in figure 3.

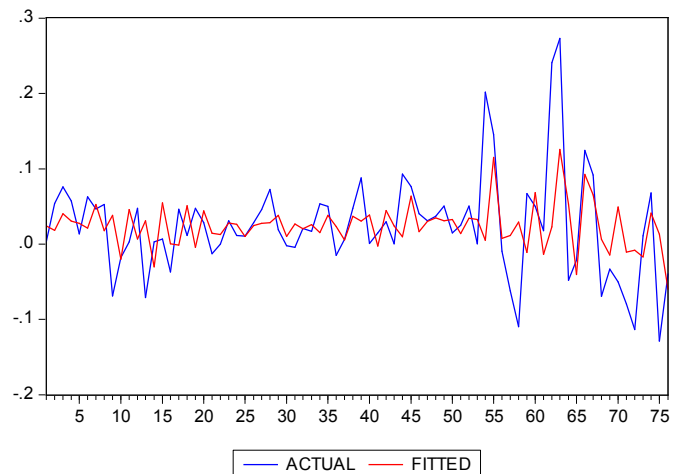


Figure 3: Prediction using AR (3) Model

The Ljung-Box statistics of standardized residuals $Q(12) = 6.0812$ with a p value 0.91 based on its asymptotic chi-squared distribution with 12 degrees of freedom. Thus, the null hypothesis of no residual serial correlation in the first 12 lags is not rejected.

Estimation of GARCH Model

The Ljung-Box statistics of the a_t^2 series shows strong conditional heteroscedasticity or ARCH effects with $Q(12) = 44.7602$, the p value of which is 1.133e-05. We can obtain the following AR(3)-IGARCH(1,1) model for the series:

$$r_t = 0.018 + 0.292r_{t-1} - 0.2256r_{t-2} + 0.205r_{t-3} + a_t, \quad a_t = \sigma_t \epsilon_t,$$

$$\sigma_t^2 = 0.046\sigma_{t-1}^2 + 0.954a_{t-1}^2,$$

$$R^2 = 0.17, \quad mse = 0.0038$$

Where r_t is the log returns. As a second method AR(3)-IGARCH(1,1) model is applied for the prediction of the log return time series and the plot of the given time series and its prediction using this method is given in figure 4.

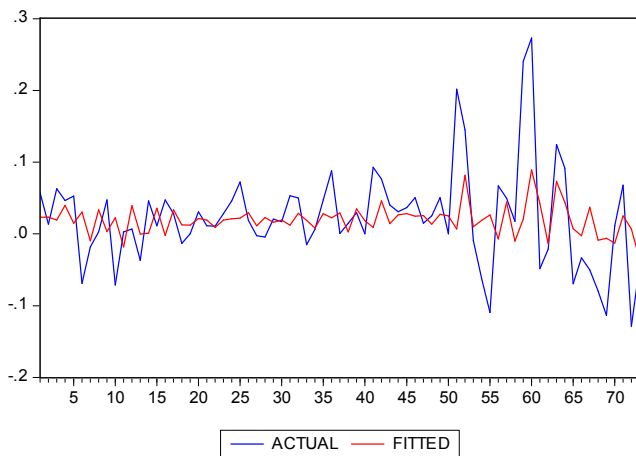


Figure 4: Prediction using AR(3)-IGARCH(1,1) Model

Linearity Testing

Table 1: Linearity Tests

Test	Tsay Test	Tlrt Test	Keenan Test
Statistics	2.553	17.848	6.699
P	0.028	0.021	0.0117

Tsay, Tlrt, and Keenan tests reject the null hypothesis that the time series follows some AR process.

Tlrt test carry out the likelihood ratio test for threshold nonlinearity, with the null hypothesis being a normal AR process and the alternative hypothesis being a TAR model with homogeneous, normally distributed errors.

Prediction using SETAR Model

The chosen SETAR model had a threshold delay d of 1 and autoregressive order 1 in the lower and 3 in the upper regime. This gives:

$$r_t = 0.0234 + 0.4742r_{t-1} + 0.0592e_t, \quad \text{if } r_{t-1} \ll 0.04988$$

$$R^2 = 0.21, \quad mse = 0.0034$$

$$r_t = -0.0260 + 0.8399r_{t-1} - 0.7969r_{t-2} + 0.8170r_{t-3} + 0.0506e_t, \quad \text{if } r_{t-1} > 0.04988$$

$$R^2 = 0.73, \quad mse = 0.0020$$

All coefficients of the model are significant at level 0.05 except intercept in the upper regime -0.0260. As a third method SETAR(2,1,3) model is applied for the prediction of the given time series and the plot of the given time series and its prediction is given in figure 5.

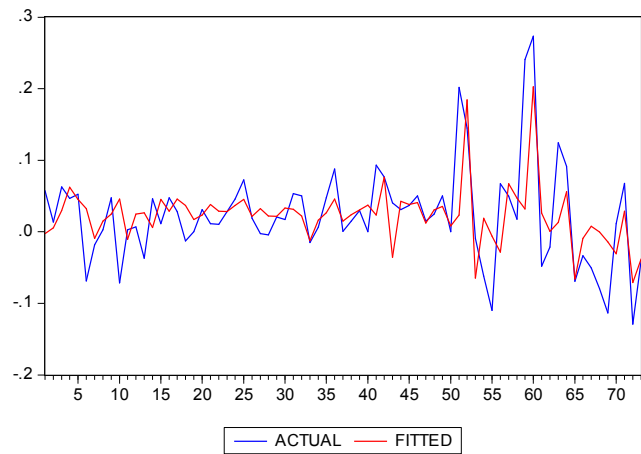


Figure 5: Prediction using SETAR(2,1,3) model

Prediction using STAR Model

Applying STAR models to the monthly log returns of gold prices, we obtain the following model:

$$r_t = 0.0279 + 0.5396r_{t-1} - 0.0238r_{t-2} - 0.0908 + \left(1 + \exp\left(-\frac{r_{t-1} - 0.0625}{67}\right)\right)^{-1} (-0.0908 + 0.5871r_{t-1} - 0.8470r_{t-2}) + a_t$$

$$AIC = -421, \quad mse = 0.0032, \quad MAPE = 858.7\%$$

All parameters in this model are not significant at level 0.05 except $\phi_1^{(2)} = -0.8470$.

COMPARISON

Comparison of the four methods considered for prediction is done by comparing their mean square error. The results of analysis are shown in table 2. Error comparison due to these four approaches given in table 2 shows that TAR (SETAR and STAR) models give smaller error. But it should be noted that the STAR model coefficients were not significant at the 0.05 level.

Table 2: Error Comparison

M.S.E(AR)	M.S.E(AR-IGARCH)	M.S.E(SETAR)	M.S.E(STAR)
0.0037	0.0038	Lower regime: 0.0034 Upper regime: 0.0020	0.0032

CONCLUSIONS

In this paper a AR model, AR-IGARCH model, SETAR and STAR models are applied for analyzing a monthly log return time series of gold price from August 2007 to November 2013 (price in Iranian Rial against 1 gram of gold). Error comparison due to these four approaches given in table 2 shows that SETAR and STAR models give smaller error. But most of the parameters of the STAR model are not significant at level 0.05. Also SETAR model gives larger R-Square than AR and AR-IGARCH models.

ACKNOWLEDGMENT

We would like to acknowledge Islamic Azad University, Dezful Branch, for financial support of the research project: "Application of Financial Time Series for forecasting Gold Price in Iran".

REFERENCES

- Ruey S. Tsay. (2010). Analysis of Financial Time Series, Third Edition, John Wiley & Sons, Inc., Hoboken, New Jersey.
- Toraman, C., Başarır, Ç., Bayramoğlu, F.M. (2011). Determination of Factors Affecting the Price of Gold: A Study of MGARCH Model, Business and Economics Research Journal, Volume 2, 4, 37-50.

- Moeini, Ali. Ahrari, Mehdi. Karimi, Parto. (2010). Forecasting Gold Price via Chaotic Models and Lyapunov Exponent, Middle Eastern Finance and Economics Journal, 8, 79-93.
- Campbell, J. Y., Lo, A. W., and MacKinlay, A. C. (1997). The Econometrics of Financial Markets. Princeton University Press, Princeton, NJ.
- Bollerslev, T. (1986). Generalised Autoregressive Conditional Heteroskedasticity, Journal of Econometrics, 31, 307-327.
- Gibson, David; and Nur, Darfiana, Threshold Autoregressive Models in Finance: A Comparative Approach, Proceedings of the Fourth Annual ASEARC Conference, 17-18 February 2011, University of Western Sydney, Paramatta, Australia.
- Tong, H., Lim, K.S., (1980) "Threshold auto regression, limit cycles, and cyclical data", Journal of the Royal Statistical Society, Vol. 42, pp 245 – 292.
- Terasvirta, T, Anderson, H M., (1992) "Characterizing Nonlinearities in Business Cycles Using Smooth Transition Autoregressive Models", John Wiley & Sons, Vol. 7.
- Nektarios Aslanidis (YE), Denise R. Osborn & Marianne Sesier (2002). Smooth transition regression models in UK stock returns, working paper.
- H. Tong. Threshold Models in Nonlinear Time Series Analysis. Springer, New York, 1983.
- M. C. Lineesh, K. K. Minu and C. Jessie John (2010). Analysis of Non-stationary Nonlinear Economic Time Series of Gold Price A Comparative Study. International Mathematical Forum, 5, 2010, no. 34, 1673 – 1683.
- McMillan, D.G (2007). Nonlinear forecasting of stock returns: Does volume help. International Journal of forecasting; 23: 115-126.
- Abken, P. (1979). The Economics of Gold Price Movements Economic Review, Federal Reserve Bank of Richmond, 3-13.
- Bauewens, L., Laurent, S. and Rombouts, J.V.K. (2003). Multivariate GARCH Models: A Survey, Core Discussion Paper, 31.

- Bollerslev, T. (1990). Modeling the Coherence in Short-Run Nominal Exchange Rate: A Multivariate Generalized ARCH Approach, *Review of Economics and Statistics*, 72, 498–505.
- Bollerslev, T., Chou, R. Y. and Kroner, K. F. (1992). ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence, *Journal of Econometrics*, 52, 5–59.
- Dooley, M.P., Isard, P. and Taylor, M.P. (1992). Exchange Rates, Country Preferences and Gold, IMF Working Paper.
- Dooley, M.P., Isard, P. and Taylor, M.P. (1995). Exchange Rates, Country Specific Shocks and Gold, *Applied Financial Economics*, 5, 121–129.
- EIA (2010), http://tonto.eia.doe.gov/dnav/pet/hist/leaf_handler.Ashx?N=Pet &S=Rbrte=M (12.05.2010)
- Enders, W. (1995). *Applied Econometric Time Series*, John Wiley & Sons Inc, Canada.
- Jacob Jaras and Azadeh M. Gishani (2010). Threshold Detection in Autoregressive Non-linear Models, Master Thesis: 15 ECTS, Lund University Department of Statistics.